Dimension and Height for Posets with Planar Cover Graphs

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Abstract

We show that for each positive integer h, there exists a least positive integer $c(h)$ so that if P is a poset having a planar cover graph and height h, then the dimension of P is at most $c(h)$. Trivially, $c(1) = 2$. In 2010, Felsner, Li, and Trotter showed that $c(2) = 4$. However, their proof techniques do not apply with h is at least 3. Here, we focus on establishing the existence of $c(h)$ for h at least 3, although we suspect that the upper bound provided by our proof is far from best possible. From below, a construction of Kelly is easily modified to show that $c(h)$ must be at least $h + 2$. This is joint work with Tom Trotter.

Keywords: poset, order dimension, cover graph.