

Zero-divisor graphs of nilpotent-free semigroups

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Abstract

To create the zero-divisor graph of a commutative semigroup (written multiplicatively) with an absorbing element 0 , let the vertices of the graph be all the nonzero zero-divisors, and draw an edge whenever the product is zero. This concept has been most intensively studied when the semigroup is the multiplicative structure of a ring, but is also useful for other ring-related semigroups. Our motivating example is the set of finite modules (up to isomorphism) over a ring, with tensor product as the product.

We find strong relationships between the zero-divisor graphs of apparently disparate kinds of nilpotent-free semigroups (i.e. those lacking any nonzero nilpotent elements) by introducing the notion of an Armendariz map between such semigroups. Such maps preserve or control several graph-theoretic invariants of the corresponding zero-divisor graphs, including those of diameter, girth, chromatic number, and clique number.

I will show how to use these maps to find relationships between the zero-divisor graphs of certain topological spaces (so-called pearled spaces), prime spectra, maximal spectra, tensor-product semigroups, and the semigroup of ideals under addition, obtaining surprisingly strong structure theorems relating ring-theoretic and topological properties to graph-theoretic invariants of the corresponding graphs.

Time permitting, I will also use our framework to demonstrate relationships between the zero-divisor graph of a ring, the zero-divisor graph of its ring of polynomials in one variable, and the annihilating-ideal graph of the ring.

This is joint work with Peyman Nasehpour.

Keywords: semigroup, zero-divisor graph, commutative ring.