Unifying Lavers Theorem and the Graph Minor Theorem via Proof System Minors

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Abstract

A graph H is a minor of a graph G if H can be obtained from G by taking subgraphs and contracting connected sets to points. The celebrated Graph Minor Theorem states that given an infinite sequence $G_1, G_2, \ldots$, of finite graphs, there exist positive integers i, j with $i < j$ such that G_i is a minor of G_j . In the language of well quasi orders, this theorem states that finite graphs are well quasi ordered under the minor relation. The similarly well-known Lavers Theorem states that scattered linear orders (i.e., those containing no suborder isomorphic to the rationals) are well quasi ordered under embedding. Though graph minors and total order embeddings appear quite disparate, in this talk we see that the notion of proof system minor includes each as a special case. This fact allows the formulation of a very general conjecture unifying the Graph Minor and Lavers Theorems. – After giving the introductory motivation, we present the definitions needed to state the unifying conjecture. We next present the conjecture itself. Since this will likely be incredibly difficult to prove in full if true, we also give various special cases of and problems motivated by the conjecture that could be attacked now. Finally, we give partial progress to date on these problems.

Keywords: graph minor, well quasi ordered poset, scattered linear orders.