

A quasisymmetric function for matroids

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Abstract

We consider the formal power series $F(M)$ defined as the weight enumerator of all generic positive integral forms over the bases of a matroid M . This association gives a Hopf algebra map from the Hopf algebra of (isomorphism classes) of matroids to QSym , the Hopf algebra of all quasisymmetric functions, displaying interesting matroid invariants when expressed in terms of the fundamental basis for QSym .

More interesting is the fact that $F(M)$ can be used to detect when the matroid polytope $P(M)$ can be decomposed into the matroid polytopes of degenerations of M . The proof of this depends on a result of Lawrence. Such decompositions have been of interest to Lafforgue in his study of compactifications of certain quotients of Grassmanians. He showed that the lack of such a decomposition implies the matroid has only a finite number of vector representations up to projective equivalence.

This is joint work with Victor Reiner and Ning Jia.

Keywords: matroid, Hopf algebra, polytope, Grassmanian manifold.