

From Efron's coins to alternation acyclic tournaments

Gabor Hetyei

The University of North Carolina at Charlotte, Charlotte, NC – 28223

Abstract

We provide necessary and sufficient conditions for a tournament to be the dominance graph of a set of unfair coins. We completely characterize the tournaments that are dominance graphs of sets of coins in which each coin displays its larger side with greater probability. The class of these tournaments coincides with the class of tournaments whose vertices can be numbered in a way that makes them semiacyclic, as defined by Postnikov and Stanley. These label the regions of the Linial arrangement. – Next we consider alternation acyclic tournaments whose set properly contains the set of semiacyclic tournaments. These label the regions in a homogenized variant of the Linial arrangement. Using a result by Athanasiadis, we show that these tournaments are counted by the median Genocchi numbers. Unexpected consequences of our results include a pair of ordinary generating function formulas for the Genocchi numbers of both kinds and a simple model for the normalized median Genocchi numbers.

Keywords: acyclic tournament, dominance graph, Linial arrangement, Genocchi numbers.