

A great tool from the polynomial method toolkit: the Alon-Füredi Theorem

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Abstract

The polynomial method is an umbrella term for a collection of algebraic statements, many of which give a bound on the number of non-zeros of a multivariate polynomial on a finite grid. One famous example is Noga Alon's Combinatorial Nullstellensatz (CN), which is a strengthening of particular case of Hilbert's Nullstellensatz. The method is growing in popularity due to the burgeoning number of applications in the fields of additive combinatorics, finite geometry, graph theory, coding theory and elsewhere. Such applications often have short and elegant proofs. – Whereas the CN gives existence statements, we study, apply and generalize a theorem of Alon and Füredi, which is tool for obtaining quantitative refinements of combinatorial existence theorems. We will show how this theorem can be used to prove and extend a classical number-theoretic result of Chevalley and Warning. We demonstrate one of the many combinatorial applications of this extension. We also discuss the theorems connection with the Schwartz-Zippel Lemma. – This talk is based on joint work with various co-authors, including Anurag Bishnoi, Pete Clark, Aden Forrow, and Aditya Potukuchi.

Keywords: polynomial mehtod, Noga Alon's Combinatorial Nullstellensatz.