Minimal ring extensions

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Abstract

Given a pair of rings $A \subset B$, B is said to be a minimal ring extension of A if there are no rings properly between A and B. Such extensions naturally divide into two cases, A integrally closed in B and B integral over A. We will discuss necessary and sufficient conditions in each of these two cases. We will also examine how the integral case subdivides further into three subcases. Then we will consider what happens with $A \cap B \subset C$ when both $A \subset C$ and $B \subset C$ are minimal ring extensions. Given time, we will also examine the extension $A \subset C$ when there exists an intermediate ring B where both $A \subset B$ and $B \subset C$ are minimal extensions. It turns out that anything is possible, from there being only finitely many intermediate rings, to there being an infinite chain of rings between A and C.

Keywords: integral extension, integrally closed, intermediate rings.