

# Self-dual embeddings of $K_{4m,4n}$ in orientable and nonorientable pseudosurfaces with the same Euler characteristic

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## Abstract

A proper embedding of a graph  $G$  in a pseudosurface  $P$  is an embedding in which the regions of the complement of  $G$  in  $P$  are homeomorphic to discs and pinchpoints of  $P$  correspond to vertices in  $G$ ; we say that a proper embedding of  $G$  in  $P$  is self dual if there exists an isomorphism from  $G$  to its topological dual. We give an explicit construction of a self-dual embedding of the complete bipartite graph  $K_{4m,4n}$  in an orientable pseudosurface for all  $m, n \geq 1$ ; we show that this embedding maximizes the number of umbrellas of each vertex and has the property that for any vertex  $v$  of  $K_{4m,4n}$ , there is a face of the constructed embedding that intersects all umbrellas of  $v$ . Leveraging these properties and applying a lemma of Bruhn and Diestel, we apply a surgery introduced here or a different known surgery of Edmonds to each of our constructed embeddings for which at least one of  $m, n \geq 2$ . The result of these surgeries is that there exist distinct orientable and nonorientable pseudosurfaces with the same Euler characteristic that feature a self-dual embedding of  $K_{4m,4n}$ . This is joint work with Justin Schroeder.

**Keywords:** graphs, graph embedding, pseudosurface.