On the Geometry of Quantum Weak Measurements

Jacob Farinholt, Naval Surface Warfare Center, Dahlgren Division, Dahlgren, VA – 22448, and George Mason University, Fairfax VA – 22030

Abstract

Weak measurements, as their name implies, only very weakly interact with the quantum system they are measuring, so as to not appreciably disturb the system. If the same weak measurement is performed on a very large ensemble of identically prepared pre- and post-selected quantum states, then the average of their measurement results yields what is known as the corresponding weak value. Curiously, this measurement result is in general a complex number. In an effort to understand the underlying mathematical structure in these measurements, we recast the notion of weak measurements in terms of well-defined linear maps from the vector space of Hermitian operators (the space of measurement observables) to the complex numbers (the weak value space). In particular, in the subspace of trace-0 Hermitian matrices, we may impart additional structure that makes this subspace a well-define Euclidean space. Consequently, by restricting our linear maps to this subspace, we may study various geometric aspects of these maps, and how they interact within the geometry of quantum mechanics. In the case of weak measurements on qubits (2-level quantum systems), we provide purely geometric conditions that are necessary and sufficient for a weak measurement to yield a purely real weak value.

Keywords: weak measurements, quantum system, complex vector space, Hermitian operator, qubits.