Applications of ordinary voltage graph theory to graph embeddability, Part II.

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Abstract

In part I, we showed that there was an infinite family of graphs not embeddable in the torus in such a way that a free action of any group on any of the graphs extends to a cellular automorphism of the torus. We used a homologically-driven analysis of ordinary voltage graph embeddings. An ordinary voltage graph embedding with voltage group A in a surface encodes a derived embedding of a derived graph in a derived surface, and the derived surface features an A-action, which is free on the graph, and is a cellular automorphism of the surface with the derived graph as the associated 1-skeleton. It is a consequence of the associated theory that if a graph embedding has the property that a group acts freely on the graph and that action extends to a cellular automorphism of the surface, then that embedding can be encoded in the form of an ordinary voltage graph embedding. – In part II, we show that for each odd prime $p > 5$, there is a ordinary voltage graph with voltage group $\mathbb{Z}_{2p}$ such the derived graph can be cellularly embedded in the nonorientable surface with Euler characteristic $2 - 2p$, but not as a derived embedding. Note that this differs from the analysis in part I where the object was to show that each derived embedding was not in a surface with fixed Euler characteristic.

Keywords: graph, group, cellular embedding, voltage graph.