Going-down in monoid rings

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Abstract

Let $A \subseteq B$ be nonzero commutative unital rings. We say that the extension satisfies the going-down property or simply going-down (GD) if given prime ideals $P' \subset P$ in A and a prime ideal Q in B such that $Q \cap A = P$, there exists a prime ideal Q' of B such that $Q' \subset Q$ and $Q' \cap A = P'$. Let S be a torsion-free cancellative abelian monoid where S has rank 1 (i.e., S can be embedded as a submonoid of $\mathbb{Q}$ under addition). Then $A[S] \subseteq B[S]$ satisfies GD if and only if the extension of polynomial rings $A[X] \subseteq B[X]$ satisfies GD. Additionally if $T \subseteq S$, then $A[T] \subseteq A[S]$ always satisfies GD. An example shows that the preceding conclusion fails if S and T each have rank 2, with A the field with two elements.

Keywords: commutative ring, prime ideal, going-down property, abelian monoid.