$$\text{Transition Matrix}: \quad T = \begin{array}{cccc} I & II & III & IV \end{array}$$

$$T = \begin{bmatrix} 1 & 0 & 0 & .02 \\ 0 & 1 & .01994 & .03 \\ 0 & 0 & .98 & 0 \\ 0 & 0 & .00006 & .95 \end{bmatrix}$$

Problem 26 (Sec 8.3) Extended

1. Suppose in a given year we have 1% of the population chronically insane, 12% mildly insane and 87% sane. What percent of the population is expected to be chronically insane after one year? After two years?

Answer: Let $X_0 = \begin{bmatrix} .01 \\ 0 \\ .87 \\ .12 \end{bmatrix}$. Calculate $T*X_0 = \begin{bmatrix} .0124 \\ .0209 \\ .8526 \\ .1141 \end{bmatrix}$ and $T^2*X_0 = \begin{bmatrix} .0147 \\ .0414 \\ .8355 \\ .0184 \end{bmatrix}$

      1.24% after one year; 1.47% after 2 years;

2. What percent of mildly insane people will be chronically insane after one year? After two years?

Answer: Use T and T^2 to get this information. Percent taken from 1st row, 4th column; 2% after one year; 3.9% after two years;

3. Suppose 200 people are mildly insane. How many of these people will be chronically insane after two years?

Answer: Let $X = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 200 \end{bmatrix}$. Calculate $T^2*X = \begin{bmatrix} 7.8 \\ 11.7 \\ 0 \\ 180.5 \end{bmatrix}$ 7.8 people

Alternatively: (use information problem 2 above) *3.9% of 200=7.8*

4. Find the percent of mildly insane people who will eventually become chronically insane.

Answer: Calculate the stable matrix for this process.
    40% of mildly insane people eventually end up chronically insane.

```
>> T=[1 0 0 .02;0 1 .01994 .03;0 0 .98 0;
0 0 .00006 .95]
T =

    1.0000       0        0    0.0200
         0   1.0000   0.0199   0.0300
         0        0   0.9800        0
         0        0   0.0001   0.9500

>> X0=[.01;0; .87;.12]
X0 =

    0.0100
         0
    0.8700
    0.1200

>> T*X0
ans =

    0.0124
    0.0209
    0.8526
    0.1141

>> T^2*X0
ans =

    0.0147
    0.0414
    0.8355
    0.1084

>> T^2
ans =

    1.0000        0   0.0000   0.0390
         0   1.0000   0.0395   0.0585
         0        0   0.9604        0
         0        0   0.0001   0.9025
```

```
>>% To find the stable matrix
>>% partition T and identify the
>>% submatrices S and R

>> S=T(1:2,3:4)

S =

         0   0.0200
    0.0199   0.0300

>> R=T(3:4,3:4)

R =

    0.9800        0
    0.0001   0.9500

>> S*inv(eye(2)-R)

ans =

    0.0012   0.4000
    0.9988   0.6000

>> stableT=[1 0 .0012 .4;0 1 .9988 .6;0
0 0 0;0 0 0 0]

stableT =

    1.0000        0   0.0012   0.4000
         0   1.0000   0.9988   0.6000
         0        0        0        0
         0        0        0        0
```