MATH 675 – HOMEWORK #1
DUE 31 JANUARY 2007

**Exercise 1.** Given Euler's formula $e^{ix} = \cos(x) + i\,\sin(x)$ derive the trig identities

$$2\,\sin(\theta)\,\sin(\phi) = \cos(\theta-\phi) - \cos(\theta+\phi),$$

$$2\,\sin(\theta)\,\cos(\phi) = \sin(\theta-\phi) + \sin(\theta+\phi).$$

**Exercise 2.** Exercise 5, page 26 in Stein and Shakarchi.

**Exercise 3.** (a) Show that if $F$ and $G$ are twice differentiable functions then the function

$$u(x,t) = F(x+t) + G(x-t) \tag{1}$$

is a solution to the wave equation $u_{tt} = u_{xx}$. This solution is known as the *travelling wave solution* to the wave equation.
(b) Show that the standing wave solution derived in class, namely

$$u(x,t) = \sum_{m=1}^{\infty} (a_m\,\cos(mt) + b_m\,\sin(mt))\,\sin(mx)$$

$$f(x) = u(x,0) = \sum_{m=1}^{\infty} a_m\,\sin(mx) \qquad g(x) = u_t(x,0) = \sum_{m=1}^{\infty} mb_m\,\sin(mx)$$

satisfies (1) with

$$F(x) + G(x) = f(x) \qquad \text{and} \qquad F'(x) - G'(x) = g(x).$$

(Hint: For these calculations you may manipulate all infinite series formally without worrying about convergence.)

**Exercise 4.** Exercise 2, page 59, Stein and Shakarchi.