
Using the table below, find the inverse Laplace transform of the function

$$F(s) = \frac{3s}{s^2 - s - 6}.$$

Solution: We need to use partial fractions to rewrite $F(s)$ in a form such that we can use the table.

$$\frac{3s}{s^2 - s - 6} = \frac{3s}{(s-3)(s+2)} = \frac{A}{s-3} + \frac{B}{s+2} = \frac{A(s+2) + B(s-3)}{(s-3)(s+2)}.$$

Therefore we must have

$$A(s+2) + B(s-3) = 3s.$$

Plugging in $s = 3$ gives $5A = 9$ or $A = 9/5$, and plugging in $s = -2$ gives $-5B = -6$ or $B = 6/5$. Therefore we have that

$$F(s) = \frac{3s}{s^2 - s - 6} = \frac{9}{5}\frac{1}{s-3} + \frac{6}{5}\frac{1}{s+2}$$

so that

$$\mathcal{L}^{-1}(F(s)) = \frac{9}{5}\mathcal{L}^{-1}\left(\frac{1}{s-3}\right) + \frac{6}{5}\mathcal{L}^{-1}\left(\frac{1}{s+2}\right).$$

consulting the table gives the answer

$$\mathcal{L}^{-1}(F(s)) = \frac{9}{5}\, e^{3t} + \frac{6}{5}\, e^{-2t}.$$

Using the table below, find the inverse Laplace transform of the function

$$F(s) = \frac{2s - 3}{s^2 - 4}.$$

Solution: We need to use partial fractions to rewrite $F(s)$ in a form such that we can use the table.

$$\frac{2s-3}{s^2 - 4} = \frac{2s-3}{(s-2)(s+2)} = \frac{A}{s-2} + \frac{B}{s+2} = \frac{A(s+2) + B(s-2)}{(s-2)(s+2)}.$$

Therefore we must have

$$A(s+2) + B(s-2) = 2s - 3.$$

Plugging in $s = 2$ gives $4A = 1$ or $A = 1/4$, and plugging in $s = -2$ gives $-4B = -7$ or $B = 7/4$. Therefore we have that

$$F(s) = \frac{2s-3}{s^2-4} = \frac{1}{4}\frac{1}{s-2} + \frac{7}{4}\frac{1}{s+2}$$

so that

$$\mathcal{L}^{-1}(F(s)) = \frac{1}{4}\mathcal{L}^{-1}\left(\frac{1}{s-2}\right) + \frac{7}{4}\mathcal{L}^{-1}\left(\frac{1}{s+2}\right).$$

consulting the table gives the answer

$$\mathcal{L}^{-1}(F(s)) = \frac{1}{4}\, e^{2t} + \frac{7}{4}\, e^{-2t}.$$

Using the table below, find the inverse Laplace transform of the function

$$F(s) = \frac{s+1}{(s-2)^2}.$$

Solution: We need to use partial fractions to rewrite $F(s)$ in a form such that we can use the table.

$$\frac{s+1}{(s-2)^2} = \frac{A}{s-2} + \frac{B}{(s-2)^2} = \frac{A(s-2)+B}{(s-2)^2}.$$

Therefore we must have

$$A(s-2) + B = s + 1.$$

Plugging in $s = 2$ gives $B = 3$, and plugging in, say, $s = 0$ gives $-2A + 3 = 1$ or $A = 1$ (I could also have taken the first derivative giving directly $A = 1$.). Therefore we have that

$$F(s) = \frac{s+1}{(s-2)^2} = \frac{1}{s-2} + \frac{3}{(s-2)^2}$$

so that

$$\mathcal{L}^{-1}(F(s)) = \mathcal{L}^{-1}\left(\frac{1}{s-2}\right) + 3\,\mathcal{L}^{-1}\left(\frac{1}{(s-2)^2}\right).$$

consulting the table gives the answer

$$\mathcal{L}^{-1}(F(s)) = e^{2t} + 3\, t\, e^{2t}.$$