MATH 214 – QUIZ 4 – SOLUTIONS

Solve the following exact equation: $(2xy^2 + 2y) + (2x^2y + 2x)y' = 0$.

Solution: We are looking for a function $\Psi(x, y)$ such that $\Psi_x = 2xy^2 + 2y$ and $\Psi_y = 2x^2y + 2x$. This leads to

$$\begin{aligned} \Psi_x &= 2xy^2 + 2y \\ \Psi &= x^2y^2 + 2xy + g(y) \\ \Psi_y &= 2x^2y + 2x + g'(y). \end{aligned}$$

This leads to

$$\begin{aligned} 2x^2y + 2x + g'(y) &= 2x^2y + 2x \\ g'(y) &= 0 \\ g(y) &= 0. \end{aligned}$$

Therefore $\Psi = x^2y^2 + 2xy$ and the solution to the ODE is given by

$$x^2y^2 + 2xy = c.$$

Solve the following exact equation: $(3x^2 - 2xy) + (6y^2 - x^2)y' = 0$.

Solution: We are looking for a function $\Psi(x, y)$ such that $\Psi_x = 3x^2 - 2xy$ and $\Psi_y = 6y^2 - x^2$. This leads to

$$\begin{aligned} \Psi_x &= 3x^2 - 2xy \\ \Psi &= x^3 - x^2y + g(y) \\ \Psi_y &= -x^2 + g'(y). \end{aligned}$$

This leads to

$$\begin{aligned} -x^2 + g'(y) &= 6y^2 - x^2 \\ g'(y) &= 6y^2 \\ g(y) &= 2y^3. \end{aligned}$$

Therefore $\Psi = x^3 - x^2y + 2y^3$ and the solution to the ODE is given by

$$x^3 - x^2y + 2y^3 = c.$$

Solve the following exact equation: $(9x^2 + y - 1) + (x - 4y)y' = 0$.

Solution: We are looking for a function $\Psi(x, y)$ such that $\Psi_x = 9x^2 + y - 1$ and $\Psi_y = x - 4y$. This leads to

$$\begin{aligned} \Psi_x &= 9x^2 + y - 1 \\ \Psi &= 3x^3 + xy - x + g(y) \\ \Psi_y &= x + g'(y). \end{aligned}$$

This leads to

$$\begin{aligned} x + g'(y) &= x - 4y \\ g'(y) &= -4y \\ g(y) &= -2y^2. \end{aligned}$$

Therefore $\Psi = 3x^3 + xy - x - 2y^2$ and the solution to the ODE is given by

$$3x^3 + xy - x - 2y^2 = c.$$