

MATH 213 – QUIZ 12 – 4 MAY 2006

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (4 pts.) Evaluate the line integral $\int_C xy \, dx + x \, dy$ where C is the curve parametrized by $x = 2t + 1$, $y = 3t$ for $0 \leq t \leq 1$.

$$\begin{aligned} x &= 2t + 1 & dx &= 2 \, dt \\ y &= 3t & dy &= 3 \, dt \end{aligned}$$

$$\int_0^1 ((2t+1)(3t)(2) + (2t+1)(3)) \, dt$$

$$= \int_0^1 (12t^2 + 12t + 3) \, dt = \left[4t^3 + 6t^2 + 3t \right]_0^1$$

$$= 4 + 6 + 3 = 13 //$$

2. (4 pts. each) Consider the planar vector field $\mathbf{F} = xy\mathbf{i} + x^2\mathbf{j}$ and the curve C given by the vector valued function $\mathbf{r}(t) = 3t^2\mathbf{i} + 2t\mathbf{j}$, $0 \leq t \leq 1$.

- (a) Calculate the *flow* of $\mathbf{F}$ along C , that is, find $\int_C \mathbf{F} \cdot \mathbf{T} \, ds$.

$$\begin{aligned} &= \int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \int_0^1 [(3t^2)(2t)\vec{\mathbf{i}} + 9t^4\vec{\mathbf{j}}] \cdot [6t\vec{\mathbf{i}} + 2\vec{\mathbf{j}}] \, dt \\ &= \int_0^1 (36t^4 + 18t^4) \, dt = \int_0^1 54t^4 \, dt = \left[\frac{54}{5}t^5 \right]_0^1 = \frac{54}{5} // \end{aligned}$$

- (b) Calculate the *flux* of $\mathbf{F}$ across C , that is, find $\int_C \mathbf{F} \cdot \mathbf{n} \, ds$.

$$\begin{aligned} &= \int_0^1 [(3t^2)(2t)\vec{\mathbf{i}} + 9t^4\vec{\mathbf{j}}] \cdot [2\vec{\mathbf{i}} - 6t\vec{\mathbf{j}}] \, dt \\ &= \int_0^1 (12t^3 - 54t^5) \, dt = \left[3t^4 - 9t^6 \right]_0^1 = -6 // \end{aligned}$$