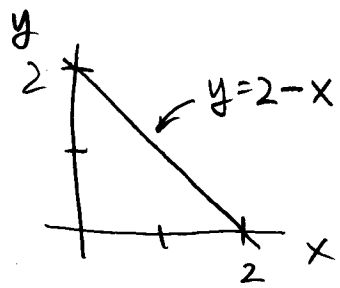
MATH 213 – QUIZ 11 – 27 APRIL 2006

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (4 pts.) Find the Jacobian, $\frac{\partial(x,y)}{\partial(u,v)}$, of the transformation $x = 2u - v^2$, $y = 2 - 2u^2$.

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 2 & -2v \\ -4u & 0 \end{vmatrix} = -8uv$$

2. (6 pts.) Use the transformation $x = u + v$, $y = u - v$ to transform the double integral $\int_0^2 \int_0^{2-x} \frac{y-x}{y+x}\, dy\, dx$ into an iterated integral in the variables u and v. DO NOT EVALUATE.

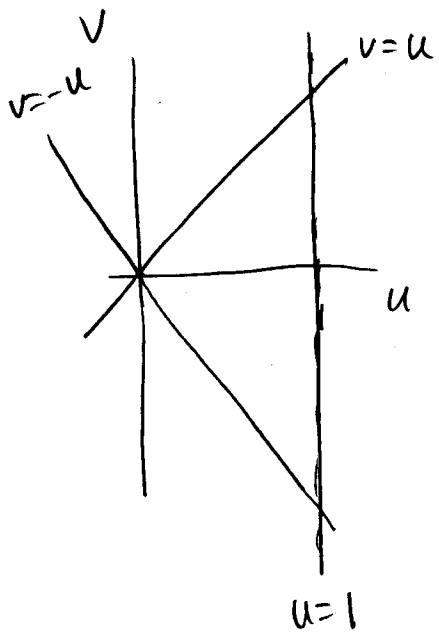


$y = 2 - x$ $\quad$ $y = 0$ $\quad$ $x = 0$

$u - v = 2 - u - v$ $\quad$ $u - v = 0$ $\quad$ $u + v = 0$

$2u = 2$ $\quad$ $v = u$ $\quad$ $v = -u$

$u = 1$



$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -2$$

$$\therefore \int_0^2 \int_0^{2-x} \frac{y-x}{y+x}\, dy\, dx = \int_0^1 \int_{-u}^{u} \frac{u-v-u-v}{u-v+u+v} \cdot |-2|\, dv\, du$$

$$= \int_0^1 \int_{-u}^{u} \frac{-2v}{2u} \cdot 2\, dv\, du = 2 \int_0^1 \int_{-u}^{u} \frac{-v}{u}\, dv\, du$$