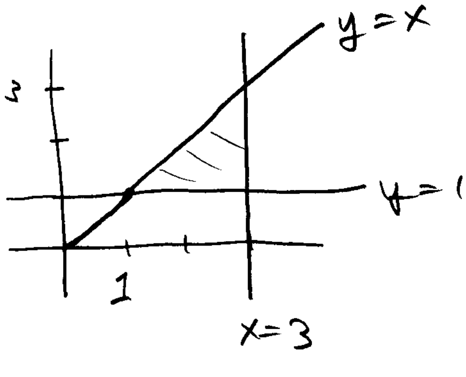
MATH 213 – QUIZ 9 – 13 APRIL 2006

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (5 pts.) Find the centroid of the triangular region bounded by the lines $y = 1$, $x = 3$, and $y = x$.



$$M_x = \int_1^3 \int_1^x y\, dy\, dx = \int_1^3 \frac{1}{2}y^2 \Big|_1^x dx$$

$$= \int_1^3 \frac{1}{2}x^2 - \frac{1}{2}\, dx = \frac{1}{6}x^3 - \frac{1}{2}x \Big|_1^3$$

$$= \left(\frac{27}{6} - \frac{3}{2}\right) - \left(\frac{1}{6} - \frac{1}{2}\right) = \frac{10}{3}$$

$$M_y = \int_1^3 \int_1^x x\, dy\, dx = \int_1^3 xy \Big|_1^x dx$$

$$= \int_1^3 (x^2 - x)\, dx = \frac{1}{3}x^3 - \frac{1}{2}x^2 \Big|_1^3$$

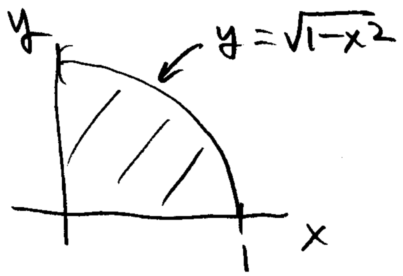
$$= \left(9 - \frac{9}{2}\right) - \left(\frac{1}{3} - \frac{1}{2}\right) = \frac{14}{3}$$

$$M = \frac{1}{2}(2)(2) = 2$$

$$\therefore \bar{x} = \frac{M_y}{M} = \frac{7}{3}$$

$$\bar{y} = \frac{M_x}{M} = \frac{5}{3} \quad //$$

2. (5 pts.) Change the integral $\int_0^1 \int_0^{\sqrt{1-x^2}} (1 + x^2 + y^2)^{-2}\, dy\, dx$ into an equivalent polar integral then evaluate the polar integral.



$$= \int_0^{\frac{\pi}{2}} \int_0^1 (1+r^2)^{-2} r\, dr\, d\theta$$

$$= \int_0^{\frac{\pi}{2}} -\frac{1}{2}(1+r^2)^{-1} \Big|_0^1 d\theta$$

$$= \int_0^{\pi/2} \left(-\frac{1}{4} + \frac{1}{2}\right) d\theta = \frac{1}{4} \cdot \frac{\pi}{2} = \frac{\pi}{8} \quad //$$