MATH 213 – QUIZ 8 – 6 APRIL 2006

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (2 pts. each) Compute the following double integrals.

(a) $\int_0^3 \int_0^2 (2x^2 + 4xy)\,dx\,dy$ $= \int_0^3 \left(\frac{2}{3}x^3 + 2x^2y \Big|_0^2\right) dy = \int_0^3 \left(\frac{16}{3} + 8y\right) dy$

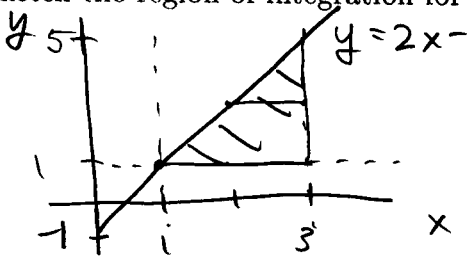
$= \frac{16}{3}y + 4y^2 \Big|_0^3 = \frac{16}{3}(3) + 4(3)^2 = 16 + 36 = 52$ //

(b) $\int_1^2 \int_0^2 \frac{y^2}{x^3}\,dy\,dx$ $= \int_1^2 \left(\frac{1}{3}\frac{y^3}{x^3}\Big|_0^2\right) dx = \int_1^2 \frac{8}{3}\cdot\frac{1}{x^3}\,dx$

$= \frac{8}{3}\cdot\frac{-1}{2}x^{-2}\Big|_1^2 = -\frac{4}{3}\left(\frac{1}{4} - 1\right) = -\frac{4}{3}\cdot\frac{-3}{4} = 1$ //

2. (2 pts. each) Consider the iterated integral $\int_1^3 \int_1^{2x-1} (xy - y^2)\,dy\,dx$.

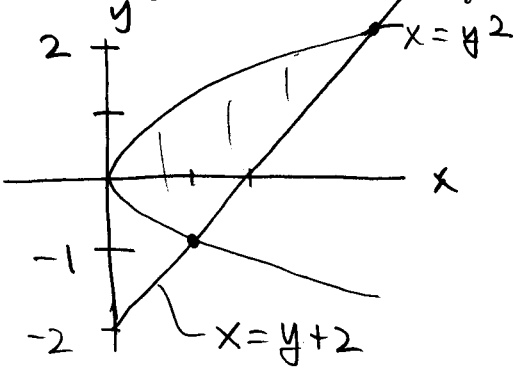
(a) Sketch the region of integration for the above integral.



(b) Interchange the order of integration for the above integral. DO NOT EVALUATE.

$$\int_1^5 \int_{\frac{y+1}{2}}^3 (xy - y^2)\,dx\,dy$$ //

3. (4 pts.) The iterated integral $\int_{-1}^2 \int_{y^2}^{y+2} dx\,dy$ represents the area of a region in the plane. Sketch the region and find its area by evaluating the given integral.



$A = \int_{-1}^2 \int_{y^2}^{y+2} dx\,dy = \int_{-1}^2 x\Big|_{y^2}^{y+2}\,dy$

$= \int_{-1}^2 y + 2 - y^2\,dy = \frac{1}{2}y^2 + 2y - \frac{1}{3}y^3 \Big|_{-1}^2$

$= 2 + 4 - \frac{8}{3} - \frac{1}{2} + 2 - \frac{1}{3} = \frac{9}{2}$ //