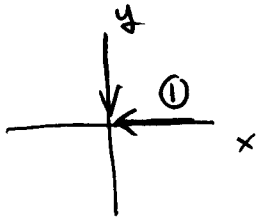
MATH 213 – QUIZ 5 – 9 MARCH 2006

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (3 pts.) Show that the limit $\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2-y}$ does not exist by finding two paths along which the limits differ.



① path is $y=0$:

$$\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2-y} = \lim_{x\to 0^+} \frac{x^2}{x^2} = 1 \;//$$

② path is $x=0$:

$$\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2-y} = \lim_{y\to 0^+} \frac{0}{0-y} = 0 \;//$$

$\therefore$ limit does not exist

2. (2 pts. each) Find the first order partial derivatives of the following functions.

(a) $f(x,y) = 4x + y^2 + 3xy$

$$\frac{\partial f}{\partial x} = 4 + 3y \;// \qquad \frac{\partial f}{\partial y} = 2y + 3x \;//$$

(b) $f(x,y) = \frac{x^2}{x^2-y}$

$$\frac{\partial f}{\partial x} = \frac{2x(x^2-y) - x^2(2x)}{(x^2-y)^2} = \frac{-2xy}{(x^2-y)^2} \;//$$

$$\frac{\partial f}{\partial y} = x^2(-1)(x^2-y)^{-2}(-1) = \frac{x^2}{(x^2-y)^2} \;//$$

3. (3 pts.) Find all second order partial derivatives (that is, f_{xx}, f_{yy}, f_{xy}, and f_{yx}) of the function $f(x,y) = \sin(xy)$.

$$f_x = y\cos(xy) \qquad f_{xx} = -y^2\sin(xy) \;//$$

$$f_y = x\cos(xy) \qquad f_{yy} = -x^2\sin(xy) \;//$$

$$f_{xy} = (f_x)_y = y(-\sin(xy)\cdot x) + \cos(xy)$$

$$= -xy\sin(xy) + \cos(xy) \;//$$

$$f_{yx} = f_{xy} \;//$$