

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. Consider the plane curve given by the vector-valued function $\mathbf{r}(t) = (1 + 2t)\mathbf{i} + t^2\mathbf{j}$.

(a) (2 pts.) Find $\mathbf{T}'(t)$ where $\mathbf{T}(t)$ is the unit tangent vector for $\mathbf{r}(t)$. (Hint: Be sure to simplify your answer as much as possible.)

$$\mathbf{r}'(t) = 2\mathbf{i} + 2t\mathbf{j} \quad |\mathbf{r}'(t)| = 2(1+t^2)^{1/2}$$

$$\mathbf{T}(t) = (1+t^2)^{-1/2} (2\mathbf{i} + 2t\mathbf{j})$$

$$\begin{aligned} \mathbf{T}'(t) &= (1+t^2)^{-1/2} (2\mathbf{j}) + (-\frac{1}{2})(1+t^2)^{-3/2} (2t) (2\mathbf{i} + 2t\mathbf{j}) \\ &= (1+t^2)^{-3/2} [(1+t^2)2\mathbf{j} - t(2\mathbf{i} + 2t\mathbf{j})] = (1+t^2)^{-3/2} (-2t\mathbf{i} + 2\mathbf{j}) // \end{aligned}$$

(a) (2 pts.) Find the curvature κ of the above curve as a function of t .

$$\kappa(t) = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{(1+t^2)^{-3/2} (1+t^2)^{1/2}}{2(1+t^2)^{1/2}} = \frac{1}{2(1+t^2)^{3/2}} //$$

(b) (3 pts.) Find the principal unit normal vector $\mathbf{N}(t)$ for the given curve.

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|} = \frac{(1+t^2)^{-3/2} (-2t\mathbf{i} + 2\mathbf{j})}{(1+t^2)^{-3/2} (1+t^2)^{1/2}} = \frac{-t}{(1+t^2)^{1/2}} \mathbf{i} + \frac{1}{(1+t^2)^{1/2}} \mathbf{j} //$$

2. (3 pts.) Find and sketch the level curve of the function $f(x, y) = 4x + y^2$ that passes through the point $(-1, 2)$.

$$f(-1, 2) = 4(-1) + (2)^2 = 0 \quad \therefore \text{level curve is } 0 = 4x + y^2$$

$$\text{or } x = -\frac{1}{4}y^2$$

