Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. Suppose that the position of a particle at time t is given by the vector-valued function $\mathbf{r}(t) = (1 + t^2)\,\mathbf{i} + t^2\,\mathbf{j} + (3 - t^2)\,\mathbf{k}$.

(a) (3 pts.) Find the velocity and acceleration of the particle.

$$\vec{r}\,'(t) = \vec{v}(t) = 2t\,\vec{i} + 2t\,\vec{j} - 2t\,\vec{k}$$

$$\vec{r}\,''(t) = \vec{a}(t) = 2\vec{i} + 2\vec{j} - 2\vec{k}$$

(b) (3 pts.) Find the speed of the particle.

$$|\vec{r}\,'(t)| = |\vec{v}(t)| = \left(4t^2 + 4t^2 + 4t^2\right)^{1/2} = \left(12t^2\right)^{1/2}$$

$$= 2\sqrt{3}\,t$$

(c) (2 pts.) Find the function $s(t)$ giving the distance travelled by the particle from the time $t = 1$ to any given time t.

$$s(t) = \int_1^t |\vec{r}\,'(\tau)|\,d\tau = \int_1^t 2\sqrt{3}\,\tau\,d\tau = \sqrt{3}\,\tau^2 \Big|_1^t$$

$$= \sqrt{3}\,t^2 - \sqrt{3} \; /\!/$$

(d) (2 pts.) Find the length of the curve given by $\mathbf{r}(t)$ for $1 \le t \le 2$.

$$L = s(2) = \sqrt{3}\,(2)^2 - \sqrt{3} = 3\sqrt{3} \; /\!/$$