MATH 213 – EXAM 3 – SOLUTIONS

1. $f(x,y) = 2x^2 + 8xy + y^4$

$$\frac{\partial f}{\partial x} = 4x + 8y \qquad -2(4x + 8y = 0)$$

$$\frac{\partial f}{\partial y} = 8x + 4y^3 \qquad 8x + 4y^3 = 0$$

$$-16y + 4y^3 = 0$$

$$y^3 - 4y = 0$$

$$y(y^2 - 4) = 0$$

$$y = 0 \quad y = 2 \quad y = -2$$

$y = 0 \rightarrow x = 0$

$y = 2 \rightarrow x = -4$

$y = -2 \rightarrow x = 4$

$\therefore$ CP: $(0,0)$ $(-4,2)$ $(4,-2)$ //

2. $f(x,y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 2$

$f_x = 6xy - 6x \qquad f_{xx} = 6y - 6 \quad f_{xy} = 6x$

$f_y = 3x^2 + 3y^2 - 6y \quad f_{yy} = 6y - 6$

$$\therefore D(x,y) = (6y-6)(6y-6) - (6x)^2$$

$$= 36(y-1)^2 - 36x^2$$

$D(0,0) = 36 > 0 \quad f_{xx}(0,0) = -6 < 0 \quad \therefore (0,0)$ loc max

$D(0,2) = 36 > 0 \quad f_{xx}(0,2) = 6 > 0 \quad \therefore (0,2)$ loc min

$D(1,1) = -36 < 0 \quad \therefore (1,1)$ saddle

$D(-1,1) = -36 < 0 \quad \therefore (-1,1)$ saddle

3. (a) $\int_0^1 \int_x^{3-x} (x+y)^2 \, dy \, dx$

$= \int_0^1 \frac{1}{3}(x+y)^3 \Big|_x^{3-x} \, dx$

$= \int_0^1 \frac{1}{3}(3^3 - (2x)^3) \, dx$

$= \int_0^1 9 - \frac{8}{3}x^3 \, dx$

$= 9x - \frac{2}{3}x^4 \Big|_0^1 = 9 - \frac{2}{3} = \frac{25}{3}$ //

(b) $\int_0^1 \int_y^{2y} \int_0^{2y-z} z \, dx \, dz \, dy$

$= \int_0^1 \int_y^{2y} xz \Big|_0^{2y-z} dz \, dy$

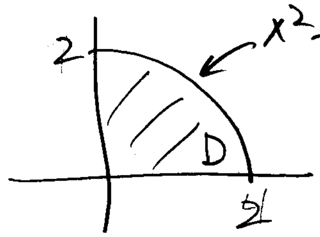
$= \int_0^1 \int_y^{2y} 2yz - z^2 \, dz \, dy$

$= \int_0^1 yz^2 - \frac{1}{3}z^3 \Big|_y^{2y} dy$

$= \int_0^1 y(2y)^2 - \frac{1}{3}(2y)^3 - y^3 + \frac{1}{3}y^3 \, dy$

$= \int_0^1 \frac{2}{3}y^3 \, dy = \frac{1}{6}y^4 \Big|_0^1 = \frac{1}{6}$ //

4.



$$\iint_D x\,dA = \int_0^{\frac{\pi}{2}}\int_0^2 r\cos\theta\, r\,dr\,d\theta$$

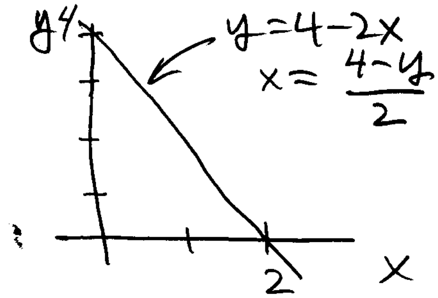
$$= \int_0^{\frac{\pi}{2}}\int_0^2 r^2\cos\theta\,dr\,d\theta = \int_0^{\frac{\pi}{2}} \frac{1}{3}r^3\cos\theta\Big|_0^2\,d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{8}{3}\cos\theta\,d\theta = \frac{8}{3}\sin\theta\Big|_0^{\frac{\pi}{2}} = \frac{8}{3}\sin\left(\frac{\pi}{2}\right)$$

$$\cancel{\frac{4\sqrt{2}}{3}} = \frac{8}{3}$$

5.

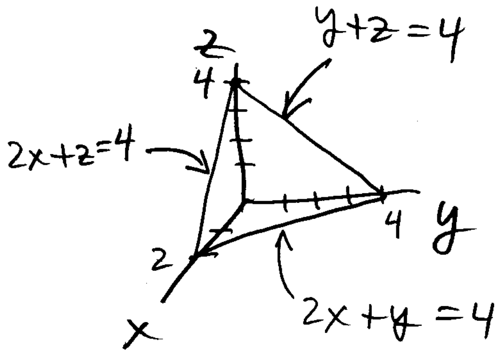
$$\int_0^2\int_0^{4-2x} xy\,dy\,dx$$



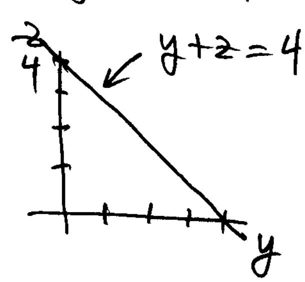
$$= \int_0^4\int_0^{\frac{4-y}{2}} xy\,dx\,dy$$

6.

$$\iiint_E xyz\,dV$$

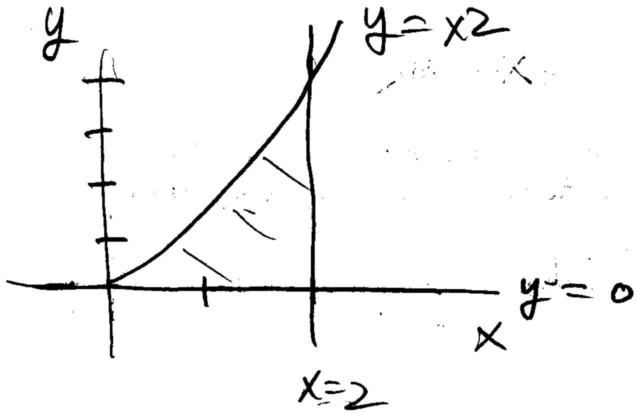


$2x+y+z=4$



$$\int_0^4\int_0^{4-z}\int_0^{\frac{1}{2}(4-y-z)} xyz\,dx\,dy\,dz$$

7.



$$M_x = \int_0^2 \int_0^{x^2} x y^2 \, dy \, dx$$

$$= \int_0^2 \frac{1}{3} x y^3 \Big|_0^{x^2} dx$$

$$= \int_0^2 \frac{1}{3} x^7 \, dx = \frac{1}{24} x^8 \Big|_0^2 = \frac{256}{24} = \frac{32}{3} \,//$$

$$M_y = \int_0^2 \int_0^{x^2} x^2 y \, dy \, dx$$

$$= \int_0^2 \frac{1}{2} x^2 y^2 \Big|_0^{x^2} dx$$

$$= \int_0^2 \frac{1}{2} x^6 \, dx = \frac{1}{14} x^7 \Big|_0^2 = \frac{128}{14} = \frac{64}{7} \,//$$

$$\therefore (\bar{x}, \bar{y}) = \left(\frac{64}{7} \cdot \frac{3}{16}, \frac{32}{3} \cdot \frac{3}{16} \right) = \left(\frac{12}{7}, 2 \right) \,//$$