MATH 213 – 25 APRIL 2006 – EXAM 3

Answer each of the following questions. Show all work, as partial credit may be given.

1. (12 pts.) Find all critical points of the function $f(x, y) = 2x^2 + 8xy + y^4$. (Hint: There are three.)

2. (12 pts.) Given that the function $f(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 2$ has critical points $(0, 0)$, $(0, 2)$, $(1, 1)$ and $(-1, 1)$, identify each as a local maximum, local minimum, or saddle point.

3. (15 pts. each) Evaluate the following iterated integrals.

(a) $\displaystyle\int_0^1 \int_x^{3-x} (x + y)^2 \, dy \, dx$

(b) $\displaystyle\int_0^1 \int_y^{2y} \int_0^{2y-z} z \, dx \, dz \, dy$

4. (12 pts.) Evaluate the integral $\displaystyle\iint_D x \, dA$ where D is the region in the first quadrant bounded by the circle $x^2 + y^2 = 4$ after changing the integral into polar coordinates.

5. (12 pts.) Change the order of integration in the integral $\displaystyle\int_0^2 \int_0^{4-2x} xy \, dy \, dx$. DO NOT EVALUATE.

6. (12 pts.) Write $\displaystyle\iiint_E xyz \, dV$ as an iterated triple integral in the order $dx \, dy \, dz$ where E is bounded by the coordinate planes and the plane $2x + y + z = 4$. DO NOT EVALUATE.

7. (12 pts.) Find the center of mass of the region in the first quadrant bounded by the lines $y = 0$, $x = 2$, and the curve $y = x^2$ when the density is given by $\delta(x, y) = xy$. (Hint: The "total mass" of the region is 16/3).