

MATH 213 – 28 MARCH 2006 – EXAM 2

Answer each of the following questions. Show all work, as partial credit may be given.

1. (5 pts. each) Consider the plane curve $\mathbf{r}(t) = 2t \mathbf{i} + t^2 \mathbf{j}$.
 - (a) Find the unit tangent vector $\mathbf{T}(t)$ for the given curve.
 - (b) Find the curvature, $\kappa(t)$, of the given curve as a function of t . (Hint: $\kappa(t) = |\mathbf{T}'(t)|/|\mathbf{r}'(t)|$.)
2. (10 pts. each) Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for each of the following functions.
 - (a) $f(x, y) = x^2 - xy^2 + 2y^3$
 - (b) $f(x, y) = \frac{x^2}{x^3 + y^3}$
 - (c) $f(x, y) = e^{-x^2} \cos(x^2 - y)$
3. (10 pts. each) Find f_{xx} , f_{yy} , f_{xy} and f_{yx} for each of the following functions.
 - (a) $f(x, y) = x^4 + 2x^2y + y^2$
 - (b) $f(x, y) = \sin(3xy)$
4. (10 pts.) Find f_{zzz} for $f(x, y, z) = e^{xyz}$.
5. (10 pts.) Suppose that the radius, r , and height, h of a right circular cylinder are changing with time, t . Find an expression for the rate of change of the volume, V , of the cylinder as a function of r , h , and their rates of change. (Hint: The volume V of a right circular cylinder of radius r and height h is $V = \pi r^2 h$.)
6. (8 pts. each) Consider the function $f(x, y, z) = xy + yz^2 + xz^3$.
 - (a) Find the gradient of f .
 - (b) Find the directional derivative of f at the point $(1, 0, 2)$ and in the direction $\mathbf{v} = \langle 2, -1, 2 \rangle$.
 - (c) Find the maximum rate of change of f at the point $(1, 0, 2)$.