## MATH 213 – 21 FEBRUARY 2006 – EXAM 1

Answer each of the following questions. Show all work, as partial credit may be given.

- 1. (6 pts. each) Let  $\mathbf{a} = 6\mathbf{i} 5\mathbf{j} + \mathbf{k}$ ,  $\mathbf{b} = \mathbf{i} + \mathbf{k}$ , and  $\mathbf{c} = \mathbf{i} + \mathbf{j} 3\mathbf{k}$ . Find each of the following.
  - (a) The angle between **a** and **b** correctly rounded to the nearest degree.
  - (b) A unit vector perpendicular to both  $\mathbf{b}$  and  $\mathbf{c}$ .
  - (c) The vector projection of **a** along **b**.
  - (d) Parametric equations for the line through the point (6, -5, 1) parallel to c.
  - (e) The equation of the plane containing the point (1, 1, -3) with normal vector **a**. Put your answer in the form Ax + By + Cz = D.
  - (f) The volume of the parallelepiped formed by the vectors  $\mathbf{a}$ ,  $\mathbf{b}$ , and  $\mathbf{c}$ . (Hint: This is  $|\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|$ .)
- 2. Let A = (2, 4, 5), B = (0, 0, 1) and C = (3, -1, 2).
  - (a) (10 pts.) Find an equation for the plane containing the points A, B and C. Put your answer in the form Ax + By + Cz = D.
  - (b) (5 pts.) Find the area of the triangle with vertices A, B, and C.

3. (10 pts.) Find parametric equations for the line of intersection of the planes given by x - y + z = 1 and 2x + y + z = 2.

4. (10 pts.) The position vector of a particle in space is given by the vector-valued function  $\mathbf{r}(t) = (t - \sin t)\mathbf{i} + (1 - \cos t)\mathbf{j} + t\mathbf{k}.$ 

- (a) Find the velocity, acceleration and speed of the particle.
- (b) Find parametric equations for line tangent to the path of the particle when t = 0.

5. (10 pts. each) Consider the curve given by the vector-valued function  $\mathbf{r}(t) = (2t+3)\mathbf{i} + (5-t^2)\mathbf{j} + 3t\mathbf{k}$ , for  $1 \le t \le 4$ .

- (a) Find the unit tangent vector for the above curve.
- (b) Set up but **DO NOT EVALUATE** an integral giving the length of the above curve.