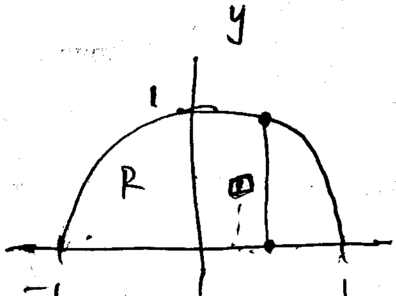
11-30-09

Exam 4 Wed. Chapter 15 (omit 15.1, 15.5, 15.6)
3x5 card allowed, no calculator (Picture ID)

Review

15.4 31 15.2 23
15.3 7 15.7 7, 9

15.2 23)



$\bar{x} = 0$

$\bar{y} =$

$y = \sqrt{1-x^2}$

$M = \text{total mass} = \frac{\pi}{2}$

$\iint_R dA = \int_{-1}^{1}\left(\int_0^{\sqrt{1-x^2}} dy\right)dx$

$M_x = \iint_R y\, dA = \int_{-1}^{1}\int_0^{\sqrt{1-x^2}} y\, dy\, dx$

moment about x-axis

$\int_{-1}^{1} \frac{1}{2}y^2\Big|_0^{\sqrt{1-x^2}} dx$

$= \int_{-1}^{1} \frac{1}{2}(1-x^2)\, dx$

$= \frac{1}{2}x - \frac{1}{6}x^3\Big|_{-1}^{1}$

$= \left(\frac{1}{2}-\frac{1}{6}\right) - \left(-\frac{1}{2}+\frac{1}{6}\right) = 1 - \frac{1}{3} = \frac{2}{3}$

Trig substitution

$x = \sin\theta$

$\int_{-1}^{1} \sqrt{1-x^2}\, dx$

$u = (1-x^2)^{1/2}$

$du = \frac{1}{2}(1-x^2)^{-1/2}(-2x)\, dx$

$= \underbrace{-x(1-x^2)^{-1/2}}_{u^{-1}} dx$

$u^2 = 1-x^2$

$x^2 = 1-u^2$

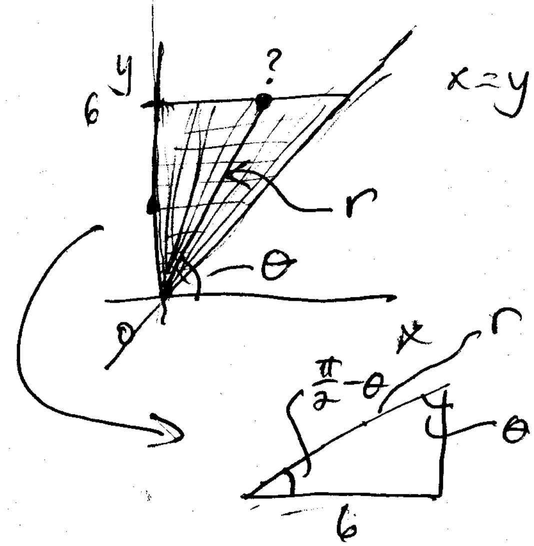
$x = \pm\sqrt{1-u^2}$

$= \frac{-\sqrt{1-u^2}}{u} dx$

$dx = \frac{-u}{\sqrt{1-u^2}} du$

$\bar{y} = \frac{M_x}{M} = \frac{2/3}{\pi/2} = \frac{4}{3\pi}$ //

15.3 7) $\displaystyle\int_0^6 \int_0^y x \, dx \, dy$



$x = r\cos\theta$
$y = r\sin\theta$

$$\int_{\pi/4}^{\pi/2} \int_{0}^{\frac{6}{\sin\theta}} r\cos\theta \, \underline{r} \, dr \, d\theta$$

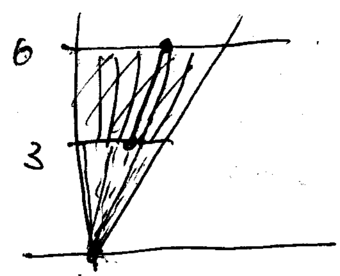
$\cos\left(\frac{\pi}{2}-\theta\right) = \frac{6}{r}$

$\sin\theta = \frac{6}{r}$

$r = \frac{6}{\sin\theta} = 6\csc\theta$

$$= \int_{\pi/4}^{\pi/2} \left(\int_0^{6/\sin\theta} r^2 \cos\theta \, dr \right) d\theta$$

e.g.



$$\int_{\pi/4}^{\pi/2} \int_{\frac{3}{\sin\theta}}^{\frac{6}{\sin\theta}} r \, dr \, d\theta$$

etc...

$$= \int_{\pi/4}^{\pi/2} \frac{1}{3} r^3 \cos\theta \Big|_{0=r}^{\frac{6}{\sin\theta}=r} d\theta$$

$$= \int_{\pi/4}^{\pi/2} \left(\frac{1}{3} \cdot \frac{6^3}{\sin^3\theta} \cdot \cos\theta - 0 \right) d\theta$$

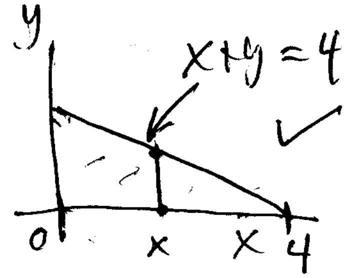
$$= \int_{\pi/4}^{\pi/2} 72 \frac{\cos\theta}{\sin^3\theta} \, d\theta$$

$u = \sin\theta$
$du = \cos\theta \, d\theta$

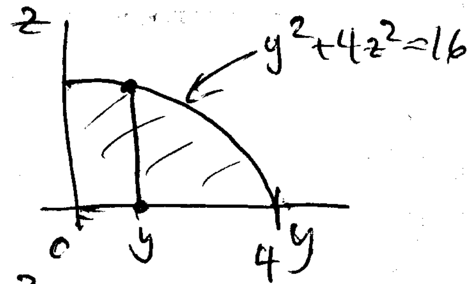
etc...

15.4 31) Find volume

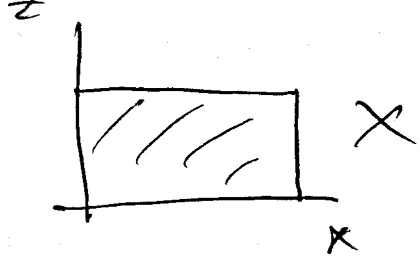
$$V = \iiint_D 1 \, dV$$



$$= \int_0^4 \int_0^{4-x} \left(\int_0^{\frac{\sqrt{16-y^2}}{2}} 1 \, \underline{dz} \right) dy\, dx$$



$$= \int_0^4 \int_0^{4-x} z \Big|_0^{\frac{1}{2}\sqrt{16-y^2}} dy\, dx$$



$$= \int_0^4 \left(\int_0^{4-x} \tfrac{1}{2}\sqrt{16-y^2}\, dy \right) dx$$

etc....

$$y^2 + 4z^2 = 16$$

$$4z^2 = 16 - y^2$$

$$z^2 = \frac{16-y^3}{4}$$

$$z = \frac{\sqrt{16-y^3}}{2}$$

$$= \int_0^4 \int_0^{\frac{\sqrt{16-y^2}}{2}} \int_0^{4-y} 1 \, dx\, dz\, dy$$

etc...

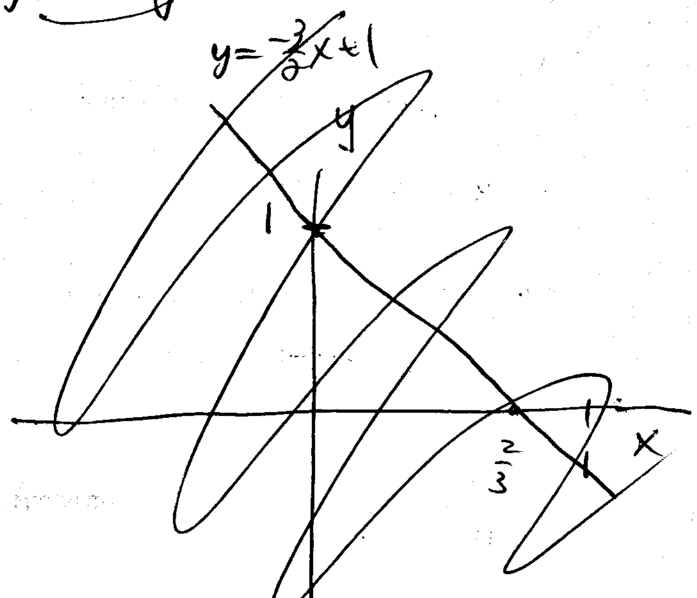
15.7 7) $\iint\limits_R (3x^2 + 14xy + 8y^2)\,dx\,dy$

$y = -\frac{3}{2}x + 1$

$y = -\frac{3}{2}x + 3$

$y = -\frac{1}{4}x$

$y = -\frac{1}{4}x + 1$



$u = 3x + 2y \quad v = x + 4y$

Get Jacobian:

$u = 3x + 2y \qquad -2(u = 3x + 2y)$

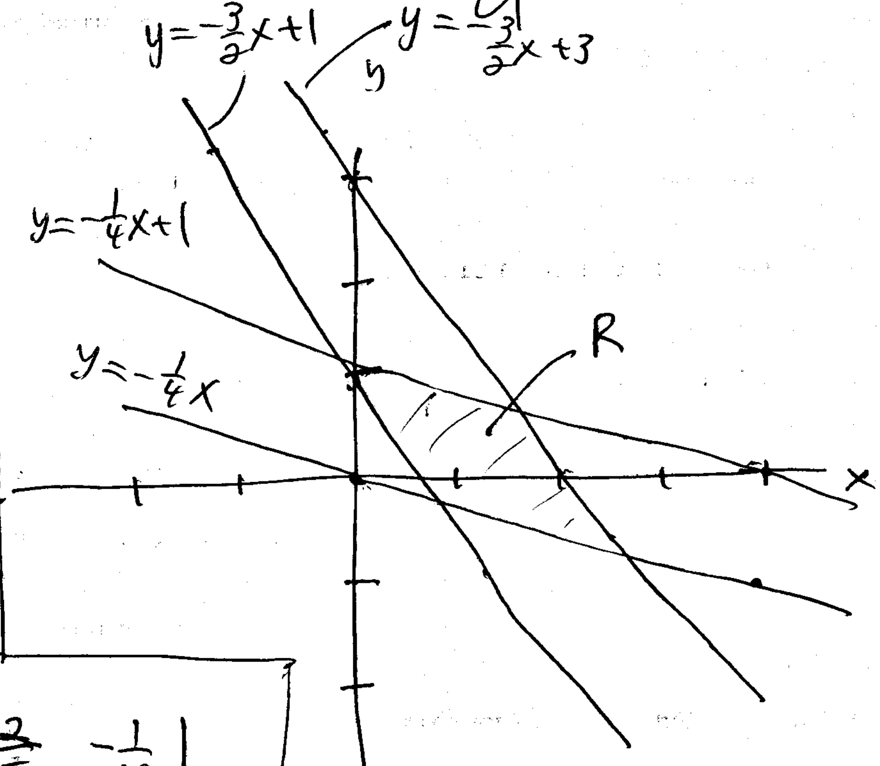
$-3(v = x + 4y) \qquad v = x + 4y$

$u - 3v = -10y \qquad -2u + v = -5x$

$y = -\frac{1}{10}(u - 3v) \qquad x = -\frac{1}{5}(-2u + v)$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} \frac{2}{5} & -\frac{1}{10} \\ -\frac{1}{5} & \frac{3}{10} \end{vmatrix}$$

$$= \frac{6}{50} - \frac{1}{50} = \frac{5}{50} = \frac{1}{10}$$



Get new region:

$$y=-\frac{3}{2}x+1 \longrightarrow -\frac{1}{10}u+\frac{3}{10}v = -\frac{3}{2}\left(\frac{2}{5}u-\frac{1}{5}v\right)+1$$

$$-\frac{1}{10}u+\cancel{\frac{3}{10}v} = -\frac{3}{5}u+\cancel{\frac{3}{10}v}+1$$

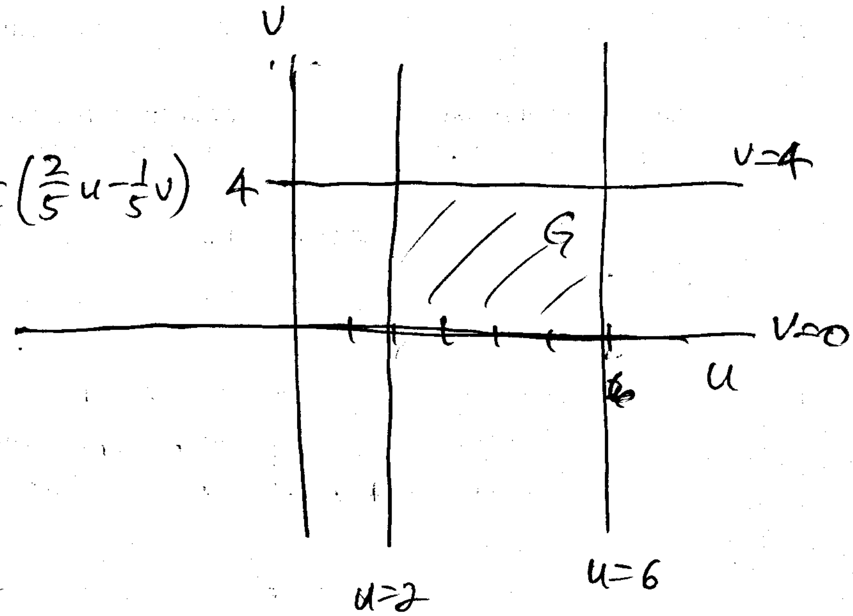
$$\frac{1}{2}u=1 \qquad u=2$$

$$y=-\frac{3}{2}x+3 \longrightarrow u=6$$

$$y=-\frac{1}{4}x \longrightarrow -\frac{1}{10}u+\frac{3}{10}v = -\frac{1}{4}\left(\frac{2}{5}u-\frac{1}{5}v\right)$$

$$-\cancel{\frac{1}{10}u}+\frac{3}{10}v = -\cancel{\frac{1}{10}u}+\frac{1}{20}v$$

$$\frac{1}{4}v=0 \qquad v=0$$



$$y=-\frac{1}{4}x+1 \longrightarrow \frac{1}{4}v=1$$

$$v=4$$

Get new integrand:

$$3x^2+14xy+8y^2 = \cancel{(3x+4y)(x+2y)}$$

$$=(3x+2y)(x+4y)$$

$$=uv$$

New integral

$$\int_0^4\int_2^6 uv\left(\frac{1}{10}\right)du\,dv$$