

MATH 203 – 9 JULY 2008 – EXAM 5

Answer each of the following questions. Show all work, as partial credit may be given. This exam is out of a total of 65 points.

1. (10 pts.) Find a basis for the subspace S spanned by $\left\{ \begin{bmatrix} 3 \\ 6 \\ -9 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ -2 \\ 5 \\ 1 \end{bmatrix} \right\}$ and state the dimension of S .

2. Let $A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$.

- (a) (5 pts.) Find $\text{rank}(A)$ and $\dim \text{Nul}(A)$.
 (b) (10 pts.) Find bases for $\text{Col}(A)$ and $\text{Row}(A)$.

3. (10 pts.) Determine if the set $\mathcal{B} = \{t^2 + 1, 1 - t^2, 1 + t\}$ is a basis for \mathbf{P}_2 , the vector space of polynomials of degree 2 or less. Fully justify your answer.

4. (10 pts. each) Let $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ -6 \end{bmatrix} \right\}$, and $\mathcal{C} = \left\{ \begin{bmatrix} 3 \\ -2 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \end{bmatrix} \right\}$, be bases for \mathbf{R}^2 , and let $\mathcal{E} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ be the standard basis.

- (a) Find the change of coordinates matrix from \mathcal{B} to the standard basis, and the change of coordinates matrix from \mathcal{C} to the standard basis.
 (b) Find the change of coordinates matrix from \mathcal{B} to \mathcal{C} and from \mathcal{C} to \mathcal{B} .

5. (10 pts.) Assuming that $\lambda = 2$ is an eigenvalue of the matrix $A = \begin{bmatrix} 0 & -4 & -6 \\ -1 & 0 & -3 \\ 1 & 2 & 5 \end{bmatrix}$, find a basis for the eigenspace of A corresponding to $\lambda = 2$.