MATH 203 – 2 JULY 2008 – EXAM 4

Answer each of the following questions. Show all work, as partial credit may be given. This exam is out of a total of 50 points.

1. (10 pts.) Let $\mathbf{H}$ be the subset of $\mathbf{P}_3$ (the set of all polynomials of degree at most 3) of polynomials of the form $\mathbf{p}(t) = a + (a+b)\,t + b\,t^2$, where a, b are in $\mathbf{R}$. Show that $\mathbf{H}$ is a subspace of $\mathbf{P}_3$ by finding a spanning set for $\mathbf{H}$.

2. (10 pts.) Let $\mathcal{B} = \left\{ \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -5 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ -7 \\ 3 \end{bmatrix} \right\}$. Show that $\mathcal{B}$ is a basis for $\mathbf{R}^3$. In other words, verify that $\mathcal{B}$ satisfies both parts of the definition of a basis.

3. Let $A = \begin{bmatrix} 1 & 0 & -5 & 1 & 4 \\ -2 & 1 & 6 & -1 & -2 \\ 0 & 2 & -8 & 2 & 12 \end{bmatrix}$.

(a) (5 pts.) Find k such that $Nul(A)$ is a subspace of $\mathbf{R}^k$ and find k such that $Col(A)$ is a subspace of $\mathbf{R}^k$.

(b) (15 pts.) Find bases for $Nul(A)$ and $Col(A)$.

4. (5 pts. each) Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ be a basis for $\mathbf{R}^2$ where $\mathbf{b}_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and $\mathbf{b}_2 = \begin{bmatrix} 5 \\ -6 \end{bmatrix}$.

(a) Find $\mathbf{x}$ if $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$.

(b) Find $[\mathbf{x}]_{\mathcal{B}}$ if $\mathbf{x} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$.