MATH 203 – 25 JUNE 2008 – EXAM 3

Answer each of the following questions. Show all work, as partial credit may be given. This exam is out of a total of 60 points.

1. Consider the linear system $A\mathbf{x} = \mathbf{b}$ where $A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 0 & 4 \\ 2 & 1 & 0 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$.

   (a) (10 pts.) Find an LU factorization of A where L is lower triangular with ones on the diagonal.

   (b) (5 pts.) Use the factorization you found in part (a) to solve the system.

   (c) (5 pts.) Use the factorization you found in part (a) to find the determinant of A.

2. (10 pts.) Find the determinant $\begin{vmatrix} 0 & 3 & 1 & 0 \\ 0 & 1 & 5 & 0 \\ -1 & 2 & 8 & 5 \\ 3 & -1 & 2 & 3 \end{vmatrix}$ using some combination of expansion in cofactors and row reduction.

3. (10 pts.) Use determinants to decide if the set of vectors $\left\{ \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ 6 \end{bmatrix}, \begin{bmatrix} 5 \\ -1 \\ -9 \end{bmatrix} \right\}$ is a linearly independent set. Fully justify your answer.

4. (10 pts.) Use Cramer's Rule to solve the linear system $\begin{bmatrix} 3 & 0 & 4 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$.

5. (10 pts.) Find the adjugate of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 2 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, and use it to find A^{-1}.