

MATH 203 – 18 JUNE 2008 – EXAM 2

Answer each of the following questions. Show all work, as partial credit may be given. This exam is out of a total of 70 points.

1. Consider the linear system
$$\begin{array}{r} x_1 + 3x_2 = 4 \\ x_1 + 4x_2 = 7 \end{array} .$$

(a) (5 pts.) Write down the *matrix equation*, $A\mathbf{x} = \mathbf{b}$, equivalent to this system and find A^{-1} .

(b) (5 pts.) Use the inverse matrix you found in part (a) to solve the system.

2. Consider the linear transformation given by $T(x_1, x_2) = (2x_2 - 3x_1, x_1 - 4x_2, x_2)$.

(a) (5 pts.) Find a matrix A such that $T(\mathbf{x}) = A\mathbf{x}$.

(b) (10 pts.) Is T one-to-one? Fully justify your answer.

(c) (10 pts.) Is the vector $\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$ in the range of T ? Fully justify your answer.

3. (5 pts. each) Let $A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 \\ 2 & -3 \\ -1 & 2 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & 2 \\ -4 & 0 \end{bmatrix}$. Indicate whether each of the following matrix products is defined, and if so then compute the product.

(a) $A^T B$

(b) $(BA)^T$

(c) BC

(d) CB

4. (5 pts. each) *Without doing any row operations*, explain why each of the following matrices is *not* invertible.

(a) $A = \begin{bmatrix} 0 & 0 & 5 \\ 1 & 0 & -1 \\ -3 & 0 & -9 \end{bmatrix}$

(b) $A = \begin{bmatrix} -3 & 4 & 6 \\ 1 & 1 & -2 \\ 3 & 2 & -6 \end{bmatrix}$

5. (10 pts.) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 3 & -5 \\ 1 & 4 & -8 \\ -3 & -7 & 10 \end{bmatrix}$.