

MATH 203 – 11 JUNE 2008 – EXAM 1

Answer each of the following questions. Show all work, as partial credit may be given. This exam is out of a total of 70 points.

$$\begin{array}{rcl} & x_1 & + 3x_2 & - 5x_3 & = & 4 \\ 1. \text{ Consider the linear system} & x_1 & + 4x_2 & - 8x_3 & = & 7 \\ & -3x_1 & - 7x_2 & + 9x_3 & = & -6 \end{array} .$$

- (a) (5 pts.) Write down the *augmented matrix* for this system.
- (b) (5 pts.) Write down the *vector equation* equivalent to this system.
- (c) (5 pts.) Write down the *matrix equation* equivalent to this system.
- (d) (10 pts.) Solve the system and describe all solutions in parametric vector form.

2. (5 pts. each) Let $A = \begin{bmatrix} -5 & 7 & 9 \\ 1 & -2 & 6 \end{bmatrix}$.

- (a) Using elementary row operations, find the *echelon form* and the *reduced echelon form* for A.
- (b) Find all solutions to the homogeneous equation $A\mathbf{x} = \mathbf{0}$ in parametric vector form.

3. Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ -2 & -4 \end{bmatrix}$.

- (a) (5 pts.) With the least amount of work possible, show that the columns of A span a plane in \mathbf{R}^3 .

- (b) (10 pts.) Is the vector $\begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$ in the span of the columns of A? Fully justify your answer.

4. (10 pts.) Do the columns of the matrix $\begin{bmatrix} 0 & 4 & 5 \\ 1 & 6 & -1 \\ -3 & 6 & -9 \end{bmatrix}$ span all of \mathbf{R}^3 ? Fully justify your answer.

5. (10 pts. each) Let $A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & -1 & 4 \\ 5 & 4 & -1 \\ -4 & -3 & 0 \end{bmatrix}$

- (a) Show that the columns of A form a linearly *dependent* set.
- (b) Find a dependence relation for the columns of A.