MATH 114 – 6 JUNE 2008 – EXAM 3

Answer each of the following questions. Show all work, as partial credit may be given.

1. (10 pts. each) Consider the cardioid given by the function $r = 2(1 + \cos(\theta))$.

(a) Set up but DO NOT EVALUATE an integral giving the area enclosed by the above curve.

(b) Set up but DO NOT EVALUATE an integral giving the length of the above curve.

(c) Find the slope of the tangent line to the above curve when $\theta = \pi/4$.

2. (10 pts. each) Apply an appropriate comparison test (that is, Direct Comparison or Limit Comparison) to determine if each of the following integrals converge or diverge. Be sure to show all work.

(a) $\displaystyle\int_0^\infty \frac{dx}{\sqrt{x^4+1}}$

(b) $\displaystyle\int_3^\infty \frac{2u^2}{3u^3+1}\,du$

3. (10 pts. each) Find the value of each of the following convergent improper integrals.

(a) $\displaystyle\int_1^\infty \frac{\ln(x)}{x^3}\,dx$ (Hint: Integrate by parts.)

(b) $\displaystyle\int_0^2 \frac{dx}{\sqrt{2-x}}$

4. (10 pts. each) Use an appropriate trigonometric substitution to evaluate each of the following integrals.

(a) $\displaystyle\int_0^1 \frac{dx}{(x^2+1)^2}$

(b) $\displaystyle\int \frac{x^2}{\sqrt{4-x^2}}\,dx$

5. (5 pts. each) Use the integral table to evaluate the following integrals.

(a) $\displaystyle\int \frac{x}{(2x+1)^{3/2}}\,dx$

(b) $\displaystyle\int x^2(1+9x^2)^{-1/2}\,dx$

$$\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x),\ \ \cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x),\ \ \sin^2(x)+\cos^2(x) = 1,\ \ \tan^2(x) = \sec^2(x) - 1$$

$$1.\ \int x(ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a^2}\left[\frac{ax+b}{n+2}-\frac{b}{n+1}\right]+C,\ n\neq -1,\ -2.$$

$$2.\ \int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}\ln(x+\sqrt{a^2+x^2})+C$$

$$3.\ \int \frac{dx}{(a^2-x^2)^2}\,dx = \frac{x}{2a^2(a^2-x^2)}+\frac{1}{4a^3}\ln\left|\frac{x+a}{x-a}\right|+C$$

$$4.\ \int \frac{\sqrt{a^2+x^2}}{x}\,dx = \sqrt{a^2-x^2}-a\ \ln\left|\frac{a+\sqrt{a^2+x^2}}{x}\right|+C$$

$$5.\ \int \frac{\sqrt{a^2+x^2}}{x^2}\,dx = \ln(x+\sqrt{a^2+x^2})-\frac{\sqrt{a^2+x^2}}{x}+C$$

$$6.\ \int \frac{x^2}{\sqrt{a^2+x^2}}\,dx = -\frac{a^2}{2}\ \ln(x+\sqrt{a^2+x^2})-\frac{x\sqrt{a^2+x^2}}{2}+C$$