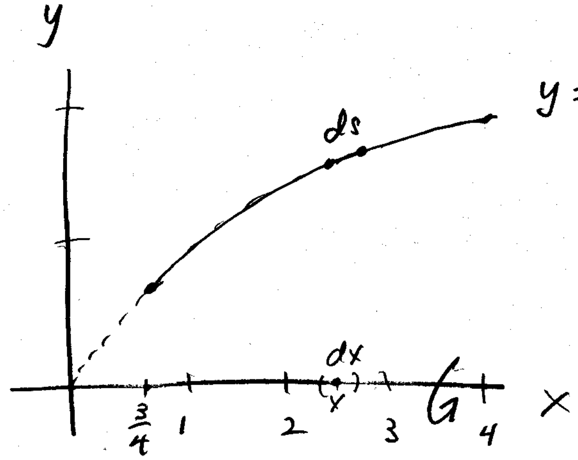
MATH 114 - EXAM 2 - SOLUTIONS

1.



$y = x^{1/2}$

$dS = 2\pi x^{1/2}\, ds$

$= 2\pi x^{1/2}\left(1+\frac{1}{4x}\right)^{1/2} dx$

$= 2\pi \left(x+\frac{1}{4}\right)^{1/2} dx$

$$S = \int_{3/4}^{4} 2\pi\left(x+\frac{1}{4}\right)^{1/2} dx$$

$ds = \left(1+\left(\frac{dy}{dx}\right)^2\right)^{1/2} dx = \left(1+\left(\frac{1}{2}x^{-1/2}\right)^2\right)^{1/2} dx$

$= \left(1+\frac{1}{4x}\right)^{1/2} dx$

2. Exponential growth $\Rightarrow$ characteristic tripling time

Population triples every 2 hours.

$\therefore$ $y(3) = 5000$

$y(5) = 5000(3) = 15000$

$y(7) = 15000(3) = 45000$

$y(9) = 45000(3) = 135000$ //

OR $y(t) = y_0 e^{kt}$

$5000 = y_0 e^{3\cdot k}$

$15000 = y_0 e^{5\cdot k}$

$\therefore$ $y_0 e^{5k} = 3 y_0 e^{3k}$

$e^{5k} = 3e^{3k}$

$e^{2k} = 3$

$2k = \ln(3)$

$k = \frac{1}{2}\ln(3)$ //

$\therefore$ $y(t) = y_0 e^{(\frac{1}{2}\ln 3)t}$

$5000 = y_0 e^{\frac{3}{2}\ln 3} = y_0 3^{3/2}$

$y_0 = 5000 \cdot 3^{-3/2} \approx 962$ //

$\therefore$ $y(t) = 962 e^{(\frac{1}{2}\ln 3)t}$

$\therefore$ $y(9) = 962 e^{\frac{9}{2}\ln 3} \approx 134965$ //

3. (a) $\int x(x^2-1)^{1/2}\,dx$ $\qquad u = x^2-1$, $du = 2x\,dx$

$= \frac{1}{2}\int 2x\,(x^2-1)^{1/2}\,dx$

$= \frac{1}{2}\int u^{1/2}\,du = \frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C$

$= \frac{1}{3}(x^2-1)^{3/2} + C$ //

(b) $\int_0^{\frac{\pi}{4}} \sec^2(x)\tan^2(x)\,dx$ $\qquad u = \tan(x)$, $du = \sec^2(x)\,dx$

$x = 0 \quad u = 0$

$x = \frac{\pi}{4} \quad u = 1$

$= \int_0^1 u^2\,du = \frac{1}{3}u^3\Big|_0^1 = \frac{1}{3}$ //

4. (a) $\int_0^{\pi} x\sin(x)\,dx$ $\qquad u = x \quad dv = \sin(x)\,dx$, $du = dx \quad v = -\cos(x)$

$= -x\cos(x)\Big|_0^{\pi} + \int_0^{\pi}\cos(x)\,dx$

$= -\pi\cos(\pi) + 0\cdot\cos(0) + \sin(x)\Big|_0^{\pi}$

$= \pi + \sin(\pi) - \sin(0) = \pi$ //

(b) $\int x^3 \ln(x)\,dx$ $\quad u = \ln(x) \quad dv = x^3 dx$

$du = \frac{1}{x} dx \quad v = \frac{1}{4}x^4$

$= \frac{1}{4}x^4 \ln(x) - \int \frac{1}{4}x^4 \cdot \frac{1}{x}\,dx$

$= \frac{1}{4}x^4 \ln(x) - \frac{1}{4}\int x^3\,dx$

$= \frac{1}{4}x^4 \ln(x) - \frac{1}{16}x^4 + C$ //

5. (a) $\left[\dfrac{2x-3}{x(x+1)^2} = \dfrac{A}{x} + \dfrac{B}{x+1} + \dfrac{C}{(x+1)^2}\right] x(x+1)^2$

$2x - 3 = A(x+1)^2 + Bx(x+1) + Cx$

$\underline{x=-1}$

$-5 = -C \quad \boxed{C=5}$

$-3 = A \quad \boxed{A=-3} \quad \underline{x=0}$

$-1 = 4A + 2B + C = -12 + 2B + 5 \quad \underline{x=1}$

$6 = 2B \quad \boxed{B=3}$

$\therefore \dfrac{2x-3}{x(x+1)^2} = \dfrac{-3}{x} + \dfrac{3}{x+1} + \dfrac{5}{(x+1)^2}$ //

(b) $\int_1^2 \frac{2x-3}{x(x+1)^2}\,dx = -3\int \frac{1}{x}\,dx + 3\int \frac{1}{x+1}\,dx + 5\int \frac{1}{(x+1)^2}\,dx$

$= -3\ln|x| + 3\ln|x+1| - 5(x+1)^{-1} \Big|_1^2$

$= -3\ln 2 + 3\ln 3 - \frac{5}{3} + 3\cancel{\ln 1}^{0} - 3\ln 2 + \frac{5}{2}$

$= -6\ln 2 + 3\ln 3 + \frac{5}{6}$ //

6. (a) $\int \sin^3(x)\cos^2(x)\,dx = \int \sin^2(x)\cos^2(x)\sin(x)\,dx$

$= \int (1-\cos^2(x))\cos^2(x)\sin(x)\,dx$ $\quad u = \cos(x)$, $du = -\sin(x)\,dx$

$= -\int (1-u^2)u^2\,du = \int (u^2-1)u^2\,du$

$= \int u^4 - u^2\,du = \frac{1}{5}u^5 - \frac{1}{3}u^3 + C = \frac{1}{5}\cos^5(x) - \frac{1}{3}\cos^3(x) + C$

(b) $\int_0^{\pi} \tan^2(x)\,dx = \int_0^{\pi}(\sec^2(x) - 1)\,dx = \tan(x) - x \Big|_0^{\pi}$

$= \cancel{\tan(\pi)}^{0} - \pi - \cancel{\tan(0)}^{0} + 0 = -\pi$ //