MATH 114 - EXAM 1 - SOLUTIONS

1 (a) $\int_0^{\pi/2} \sin^4(x)\cos(x)\,dx$ $\qquad u = \sin(x)$, $du = \cos(x)\,dx$

$= \int_0^1 u^4\,du = \frac{1}{5}u^5 \Big|_0^1$ $\qquad x=0 \quad u=0$, $x=\frac{\pi}{2} \quad u=1$

$= \frac{1}{5}$ //

(b) $\int_0^1 \theta(1-\theta)^{1/2}\,d\theta$ $\qquad u = 1-\theta \quad \theta = 1-u$, $du = -d\theta$

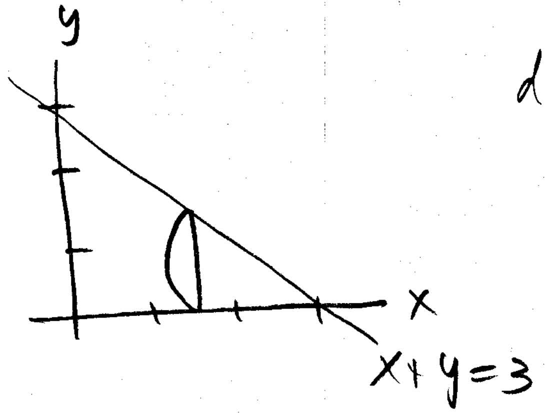
$\theta = 0 \quad u = 1$

$= -\int_1^0 (1-u)u^{1/2}\,du$ $\qquad \theta = 1 \quad u = 0$

$= \int_0^1 u^{1/2} - u^{3/2}\,du = \frac{2}{3}u^{3/2} - \frac{2}{5}u^{5/2} \Big|_0^1$

$= \frac{2}{3} - \frac{2}{5} = \frac{4}{15}$ //

2.

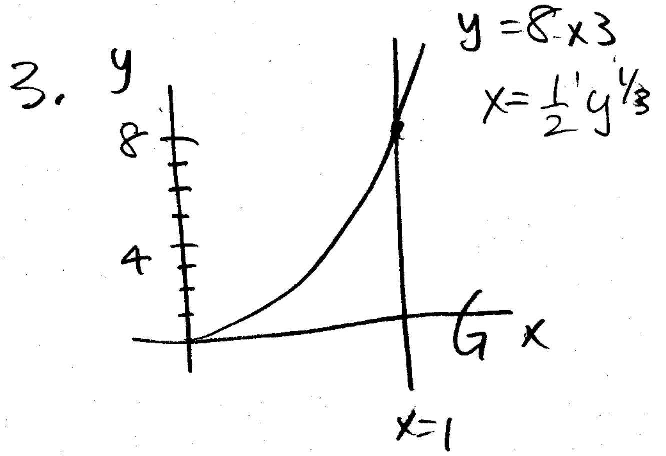


$$dV = A(x)\,dx = \frac{\pi}{2}\left(\frac{3-x}{2}\right)^2 dx$$

$$V = \int_0^3 \frac{\pi}{8}(9-6x+x^2)\,dx$$

$$= \frac{\pi}{8}\left(9x - 3x^2 + \frac{1}{3}x^3 \Big|_0^3\right) = \frac{\pi}{8}(27-27+9) = \frac{9\pi}{8} \,//$$

3.



(a) $dV = \pi(8x^3)^2\,dx$

$$V = \int_0^1 64\pi x^6\,dx$$

$$= 64\pi \cdot \frac{1}{7}x^7 \Big|_0^1 = \frac{64\pi}{7} \,//$$

(b) $dV = 2\pi x(8x^3)\,dx$

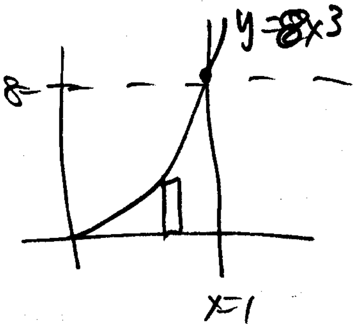
$$V = \int_0^1 16\pi x^4\,dx = 16\pi \cdot \frac{1}{5}x^5 \Big|_0^1 = \frac{16\pi}{5} \,//$$

(c) $dV = \pi\left(1^2 - \left(\frac{1}{2}y^{1/3}\right)^2\right)dy$

$$V = \int_0^8 \pi\left(1 - \frac{1}{4}y^{2/3}\right)dy \,// = \pi\left(y - \frac{3}{20}y^{5/3}\right)\Big|_0^8$$

$$= \pi\left(8 - \frac{3}{20}8^{5/3}\right) = \pi\left(8 - \frac{3}{20}\cdot 32\right) = \frac{16\pi}{5} \,//$$

(d) <u>Washers:</u>



$$dV = \pi\left(8^2 - (8-8x^3)^2\right)dx$$

$$= \pi\left(64 - (64 - 128x^3 + 64x^6)\right)dx$$

$$= \pi\left(128x^3 - 64x^6\right)dx$$

$$V = \int_0^1 \pi(128x^3 - 64x^6)\,dx$$

$$= \pi\left(32x^4 - \frac{64}{7}x^6 \Big|_0^1\right) = \pi\left(32 - \frac{64}{7}\right) = \frac{768\pi}{7}$$

4. $ds = \left(1 + \left(\frac{dy}{dx}\right)^2\right)^{1/2} dx \qquad y = x^{1/2} - \frac{1}{3}x^{3/2} \quad 1 \le x \le 4$

$$= \left(1 + \frac{1}{4x} - \frac{1}{2} + \frac{x}{4}\right)^{1/2} dx \qquad \frac{dy}{dx} = \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{1/2}$$

$$= \left(\frac{1}{4x} + \frac{1}{2} + \frac{x}{4}\right)^{1/2} dx \qquad \left(\frac{dy}{dx}\right)^2 = \frac{1}{4}(x^{-1} - 2 + x)$$

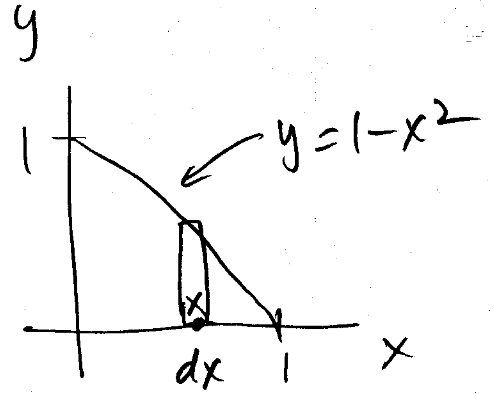
$$= \left(\left(\frac{1}{2}x^{-1/2} + \frac{1}{2}x^{1/2}\right)^2\right)^{1/2} dx$$

$$= \left(\frac{1}{2}x^{-1/2} + \frac{1}{2}x^{1/2}\right) dx$$

$$L = \int_1^4 \frac{1}{2}x^{-1/2} + \frac{1}{2}x^{1/2}\,dx = \frac{1}{2}\left(2x^{1/2} + \frac{2}{3}x^{3/2}\right)\Big|_1^4$$

$$= \frac{1}{2}\left(2\cdot 2 + \frac{16}{3} - 2 - \frac{2}{3}\right) = \frac{1}{2}\left(2 + \frac{14}{3}\right) = \frac{10}{3} \;//$$

5.



(a) $M_x = \frac{1}{2}\int_0^1 (1-x^2)^2\,dx = \frac{1}{2}\int_0^1 (1-2x^2+x^4)\,dx$

$$\boxed{dm_x = \left(\frac{1-x^2}{2}\right)dm = \left(\frac{1-x^2}{2}\right)(1-x^2)\,dx}$$

$= \frac{1}{2}\left(x - \frac{2}{3}x^3 + \frac{1}{5}x^5\right)\Big|_0^1 = \frac{1}{2}\left(1 - \frac{2}{3} + \frac{1}{5}\right) = \frac{4}{15}$ //

(b) $M_y = \int_0^1 x(1-x^2)\,dx = \int_0^1 (x - x^3)\,dx$

$$\boxed{dm_y = x(1-x^2)\,dx}$$

$= \left(\frac{1}{2}x^2 - \frac{1}{4}x^4\right)\Big|_0^1 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$ //

(c) $M = \int_0^1 (1-x^2)\,dx = x - \frac{1}{3}x^3\Big|_0^1 = 1 - \frac{1}{3} = \frac{2}{3}$ //

$\bar{x} = \frac{M_y}{M} = \frac{\frac{1}{4}}{\frac{2}{3}} = \frac{3}{8}$ $\bar{y} = \frac{M_x}{M} = \frac{\frac{4}{15}}{\frac{2}{3}} = \frac{\cancel{4}^{2}\cdot\cancel{3}}{\cancel{15}_{5}\cdot\cancel{2}} = \frac{2}{5}$

$(\bar{x}, \bar{y}) = \left(\frac{3}{8}, \frac{2}{5}\right)$ //