

>
MAPLE demo on Simpsons rule

> **f:=t->t^3+t;**

$$f := t \rightarrow t^3 + t$$

> **A:=int(f(x),x=0..1);**

$$A := \frac{3}{4}$$

> **with(student);**

[D, Diff, Doubleint, Int, Limit, Lineint, Product, Sum, Tripleint, changevar,
completesquare, distance, equate, integrand, intercept, intparts, leftbox, leftsum,
makeproc, middlebox, middlesum, midpoint, powsubs, rightbox, rightsum,
showtangent, simpson, slope, summand, trapezoid]

> **A1:=simpson(f(x),x=0..1,4);**

$$A1 := \frac{1}{6} + \frac{1}{3} \left(\sum_{i=1}^2 \left(\left(\frac{i}{2} - \frac{1}{4} \right)^3 + \frac{i}{2} - \frac{1}{4} \right) \right) + \frac{1}{6} \left(\sum_{i=1}^1 \left(\frac{1}{8} i^3 + \frac{1}{2} i \right) \right)$$

> **evalf(%);**

$$0.7500000000$$

Simpsons evaluates this one exactly since error estimate is zero.

Lets try different example.

> **g:=x->exp(-x^2);**

$$g := x \rightarrow e^{(-x^2)}$$

> **A:=int(g(x),x=0..2);**

$$A := \frac{1}{2} \operatorname{erf}(2) \sqrt{\pi}$$

> **evalf(%);**

$$0.8820813910$$

> **A1:=simpson(g(x),x=0..2,4);**

$$A1 := \frac{1}{6} + \frac{1}{6} e^{(-4)} + \frac{2}{3} \left(\sum_{i=1}^2 e^{(-(i-1/2)^2)} \right) + \frac{1}{3} \left(\sum_{i=1}^1 e^{(-i^2)} \right)$$

> **evalf(%);**

$$0.8818124254$$

> **E1:=evalf(abs(A1-A));**

$$E1 := 0.0002689656$$

```

> A2:=evalf(simpson(g(x),x=0..2,8));
      A2 := 0.8820655104

> E2:=evalf(abs(A2-A));
      E2 := 0.0000158806

> E1/E2;
      16.93674043

```

Comparison with trapezoid rule.

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> B1:=evalf(trapezoid(g(x),x=0..2,4));
      B1 := 0.8806186342

> ET1:=evalf(abs(B1-A));
      ET1 := 0.0014627568

> B2:=evalf(trapezoid(g(x),x=0..2,8));
      B2 := 0.8817037914

> ET2:=evalf(abs(B2-A));
      ET2 := 0.0003775996

```

Trapezoid worse than Simpson for same n and also decreases less rapidly.

Demo of partial fraction expansions.

```

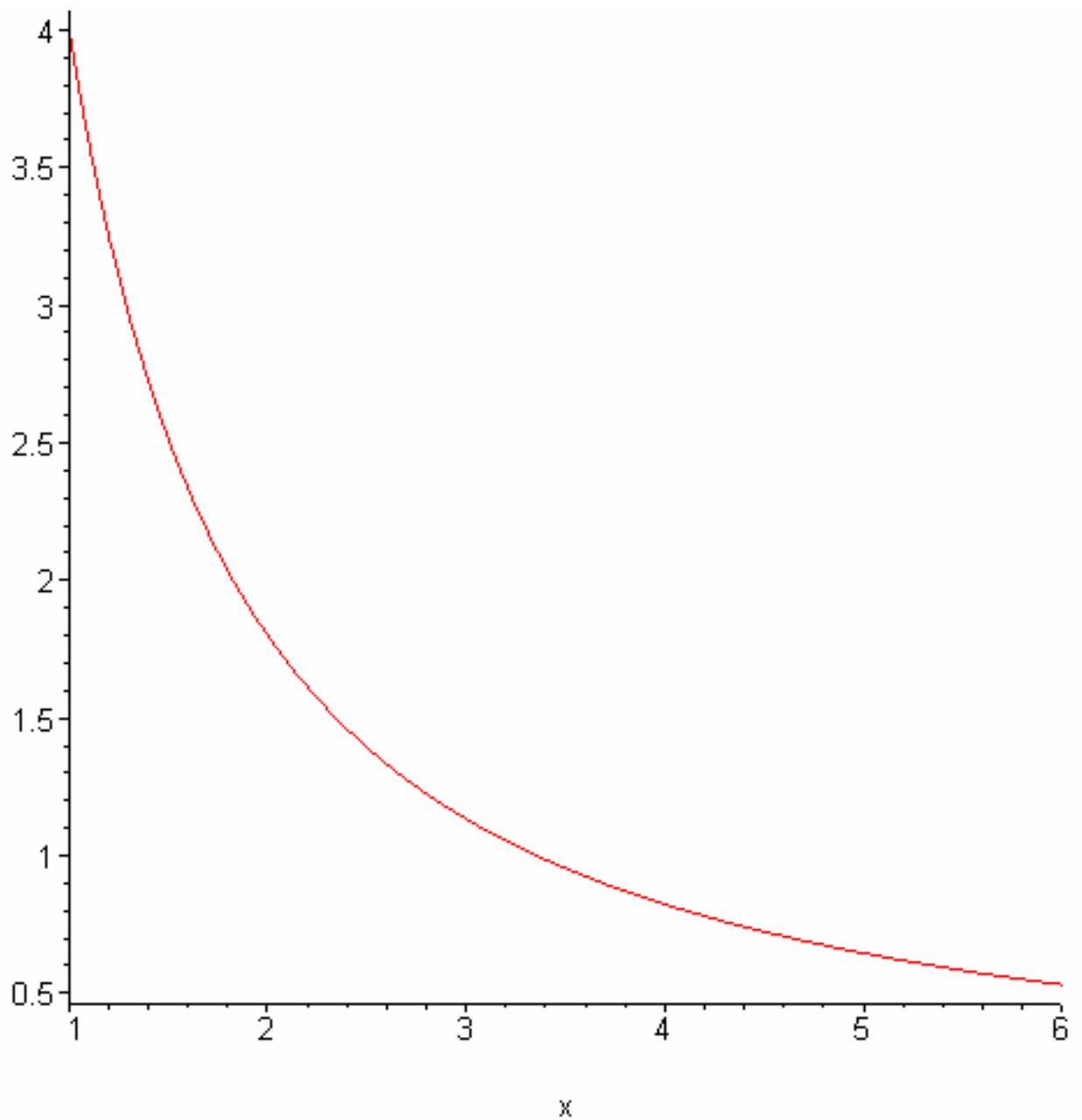
> help(parfrac);
> f:=x->(3*x^2+x+4)/(x*(x^2+1));
      f := x →  $\frac{3x^2 + x + 4}{x(x^2 + 1)}$ 

> f(2);
       $\frac{9}{5}$ 

> f(3);
       $\frac{17}{15}$ 

> plot(f(x),x=1..6);

```



> `convert(f(x), parfrac, x);`

$$\frac{4}{x} + \frac{-x+1}{x^2+1}$$

> `int(f(x), x);`

$$-\frac{1}{2} \ln(x^2+1) + \arctan(x) + 4 \ln(x)$$

>