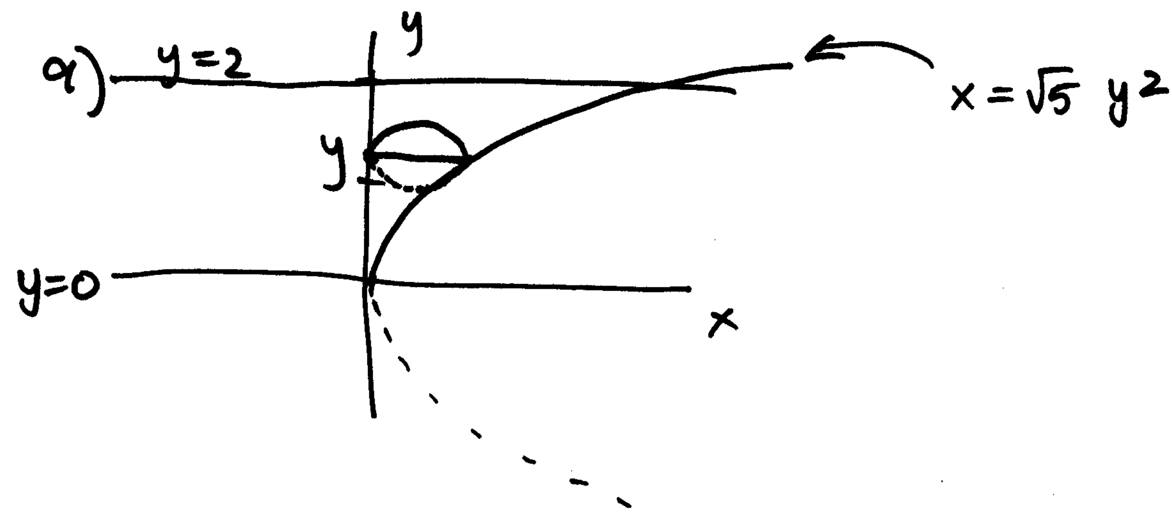
MONDAY - WE ARE IN ENTERPRISE 80

EXAM 1 - 6.1-6.4   MAPLE 1 - OUT MONDAY

Review 6.1 17, 35, 45, 55, 9   6.4 23, 25
6.3 28, 29   6.2 17

6.1 9)



Find volume.

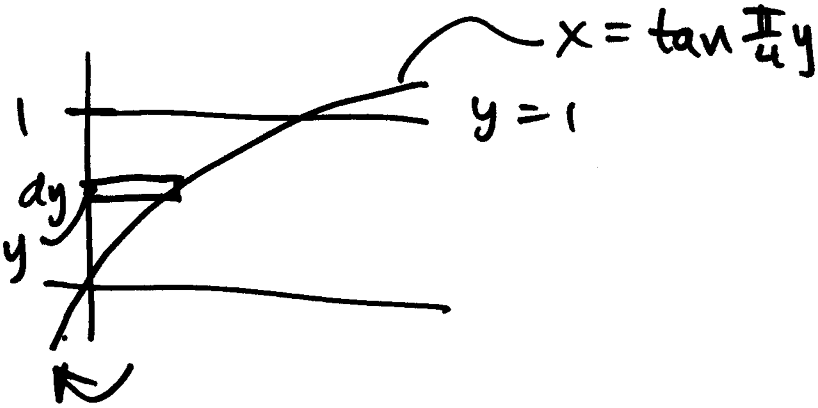
$$dV = (\text{area of cross section})\,dy = A(y)\,dy$$

$$= \pi\left(\frac{\sqrt{5}\,y^2}{2}\right)^2 dy = \pi\,\frac{5}{4}\,y^4\,dy$$

$$V = \int_0^2 dV = \int_0^2 \frac{5\pi}{4}\,y^4\,dy = \frac{5\pi}{4}\int_0^2 y^4\,dy$$

$$= \frac{5\pi}{4}\cdot\frac{1}{5}y^5\Big|_0^2 = \frac{\pi}{4}(32-0) = \frac{32\pi}{4} = 8\pi$$

17)



$$dV = \pi(\text{radius})^2\,dy$$

$$= \pi\left(\tan\tfrac{\pi}{4}y\right)^2 dy = \pi\tan^2\left(\tfrac{\pi}{4}y\right)dy$$

$$V = \int_0^1 \pi\tan^2\left(\tfrac{\pi}{4}y\right)dy$$

$$\frac{\cos^2(x)}{\cos^2(x)} + \frac{\sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)}$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$\tan^2(x) = \sec^2(x) - 1$$

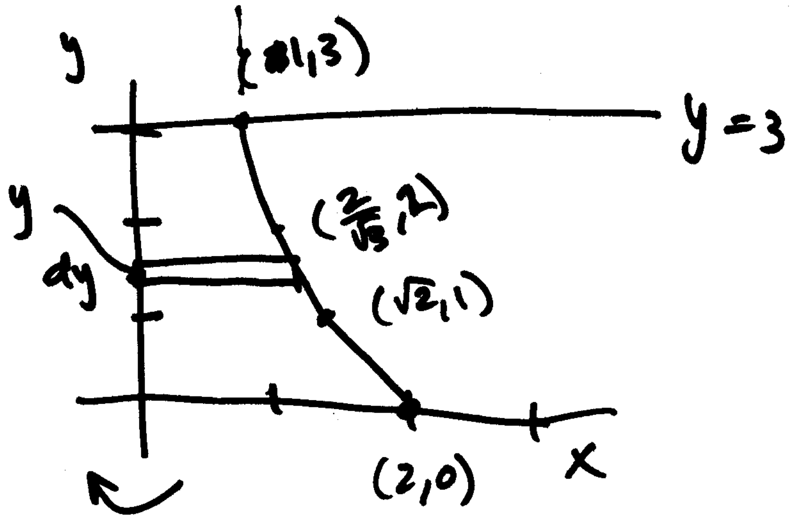
$$= \pi\int_0^1\left(\sec^2\left(\tfrac{\pi}{4}y\right) - 1\right)dy$$

$$= \pi\left(\tfrac{4}{\pi}\tan\left(\tfrac{\pi}{4}y\right) - y\Big|_0^1\right)$$

$$= \pi\left(\tfrac{4}{\pi}\tan\left(\tfrac{\pi}{4}\right) - 1 - \tfrac{4}{\pi}\cancel{\tan(0)}^{0} + 0\right)$$

$$= \pi\left(\tfrac{4}{\pi} - 1\right) = 4 - \pi \,//$$

35)



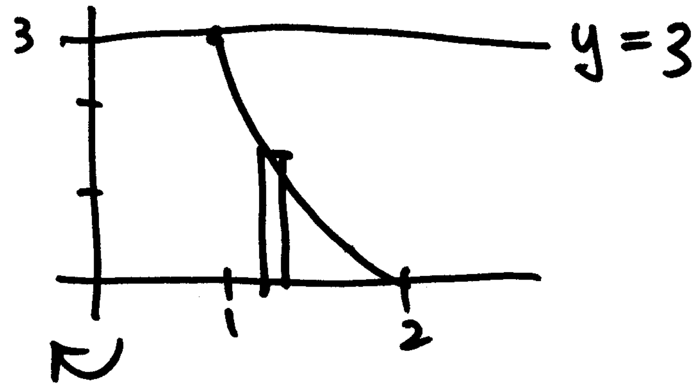
$$x = \frac{2}{(y+1)^{1/2}}$$

$$dV = \pi\left(\frac{2}{(y+1)^{1/2}}\right)^2 dy = \pi \cdot \frac{4}{y+1}\,dy$$

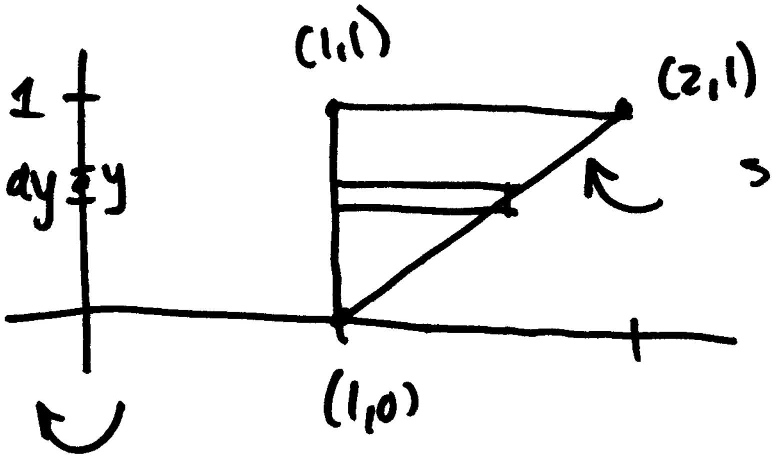
$$V = \int_0^3 \frac{4\pi}{y+1}\,dy = 4\pi\int_0^3 \frac{dy}{y+1}$$

$$= 4\pi \ln(y+1)\Big|_0^3 = 4\pi(\ln 4 - \ln 1) = 4\pi \ln 4$$

shells:



45)



slope = 1

$y - 0 = 1(x-1)$

$y = x - 1$

$x = y + 1$

$$dV = \pi\left(\begin{pmatrix}\text{outer}\\ \text{rad.}\end{pmatrix}^2 - \begin{pmatrix}\text{inner}\\ \text{rad.}\end{pmatrix}^2\right) dy$$

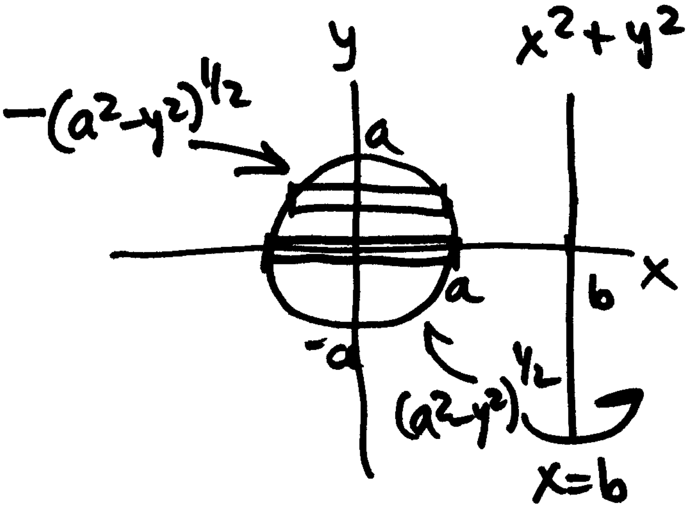
$$= \pi\left((y+1)^2 - 1^2\right) dy$$

$$= \pi\left(y^2 + 2y + 1 - 1\right) dy$$

$$= \pi\left(y^2 + 2y\right) dy$$

$$V = \int_0^1 \pi\left(y^2 + 2y\right) dy \quad \text{etc}$$

55)



$x^2 + y^2 \leq a^2$

$x^2 + y^2 = a^2$

$x = \pm\sqrt{a^2 - y^2}$

Washers:

$$dV = \pi\left(\begin{pmatrix}\text{outer}\\ \text{rad}\end{pmatrix}^2 - \begin{pmatrix}\text{inner}\\ \text{rad}\end{pmatrix}^2\right) dy$$

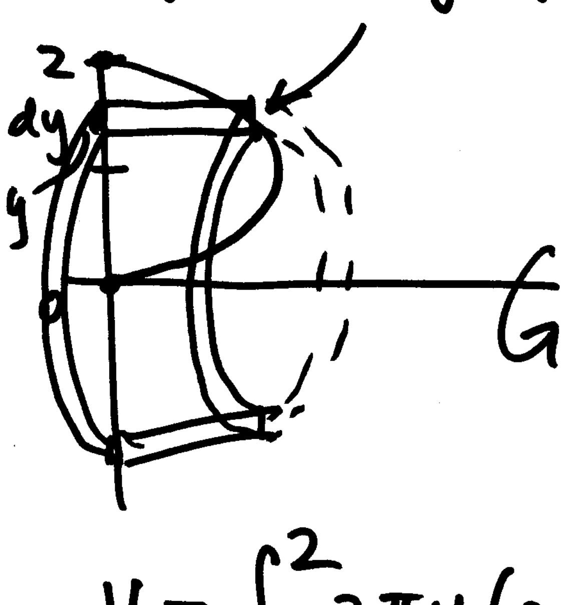
$$= \pi\left(\left(b + \left(+(a^2-y^2)^{1/2}\right)\right)^2 - \left(b - (a^2-y^2)^{1/2}\right)^2\right) dy$$

$$= \pi\Big(\cancel{b^2} + 2b(a^2-y^2)^{1/2} + \cancel{(a^2-y^2)}$$
$$- \left(\cancel{b^2} - 2b(a^2-y^2)^{1/2} + \cancel{(a^2-y^2)}\right)\Big) dy$$

$$= \pi\left(4b(a^2-y^2)^{1/2}\right) dy$$

$$V = \int_{-a}^{a} 4\pi b (a^2-y^2)^{1/2} dy \quad \cdots$$

6.2 17) $x = 2y - y^2$, $x = 0$



$$0 = 2y - y^2$$
$$0 = y(2-y)$$

x $dV = 2\pi(\text{radius})(\text{height})\, dy$

$$= 2\pi y (2y - y^2) dy$$

$$V = \int_0^2 2\pi y (2y - y^2) dy$$

6.3 29) $L = \int_1^4 \sqrt{1 + \frac{1}{4x}}\, dx$

$$L = \int_1^4 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

= length of curve $y = f(x)$
$1 \le x \le 4$

$\therefore \frac{dy}{dx} = \sqrt{\frac{1}{4x}} = \frac{1}{2\sqrt{x}} = \frac{1}{2} x^{-1/2}$

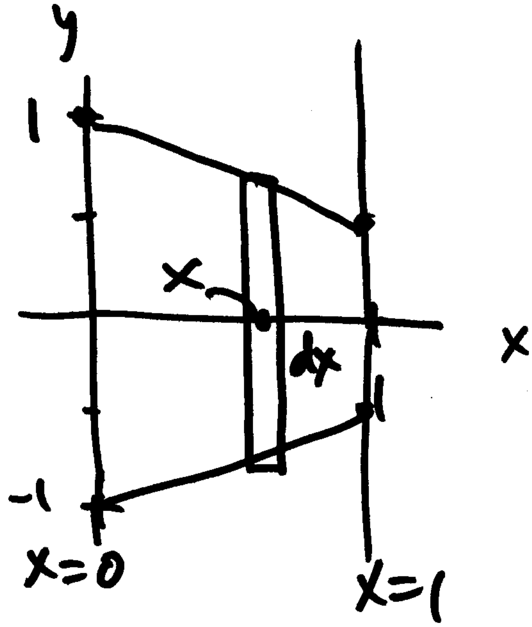
$$y = \frac{1}{2} \cdot 2x^{\frac{1}{2}} + C = x^{1/2} + C$$

Curve passes through (1,1) we have

$1 = (1)^{1/2} + c$ or $c = 0$

$y = x^{1/2}$ is the curve

6.4   23)



$$y = \frac{1}{1+x^2}$$

$$y = -\frac{1}{1+x^2}$$

Find c.o.m. $(\bar{x}, \bar{y})$

density $= \delta =$ const.

Know $\bar{y} = 0$ by symmetry.

Need $\bar{x}$.

$$\bar{x} = \frac{M_y}{M}$$

$M = \delta$ (area between curves)

$$= \delta \int_0^1 \frac{1}{1+x^2} - \left(-\frac{1}{1+x^2}\right) dx$$

$$= \delta \int_0^1 \frac{2}{1+x^2}\, dx = 2\delta \tan^{-1}(x)\Big|_0^1$$

$$= 2\delta\left(\tan^{-1}(1) - \tan^{-1}(0)\right)$$

$$= 2\delta\left(\frac{\pi}{4} - 0\right) = \frac{\pi\delta}{2}$$

M_y:

$$dm = \delta\left(\frac{2}{1+x^2}\right)dx$$

$$M_y = \int_0^1 x\, dm = \int_0^1 x \cdot \frac{2\delta}{1+x^2}\, dx$$

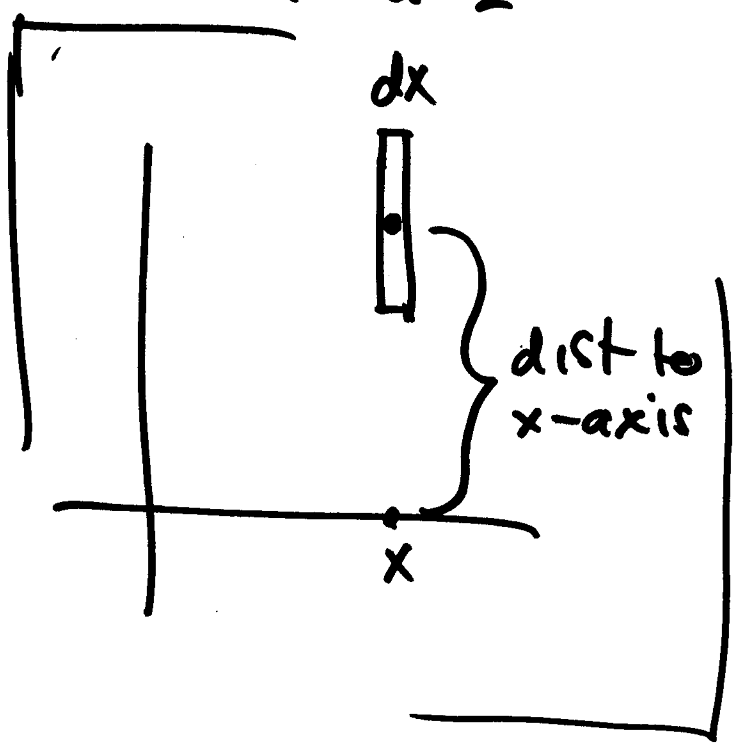
$$= \frac{2\delta}{2}\int_0^1 \frac{2x}{1+x^2}\, dx$$

$u = 1+x^2$

$du = 2x\, dx$

$x=0 \quad u=1$

$x=1 \quad u=2$



$$= \delta \int_1^2 \frac{du}{u}$$

$$= \delta \ln(u)\Big|_1^2$$

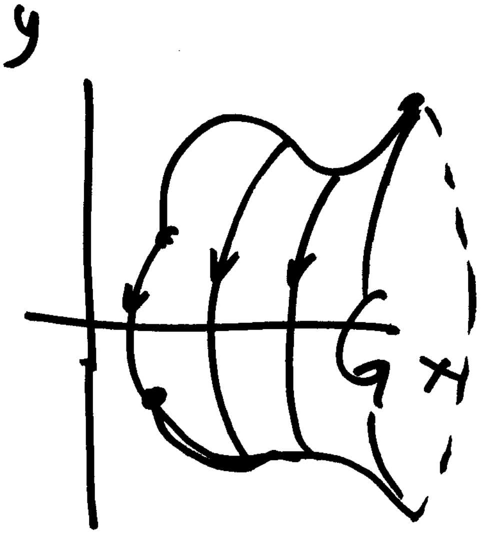
$$= \delta\left(\ln 2 - \cancel{\ln 1}^{\,0}\right)$$

$$= \delta \ln(2) \,//$$

$$\bar{x} = \frac{M_y}{M} = \frac{\cancel{\delta}\ln 2}{\frac{\pi \cancel{\delta}}{2}} = \frac{2\ln 2}{\pi} \approx .44$$

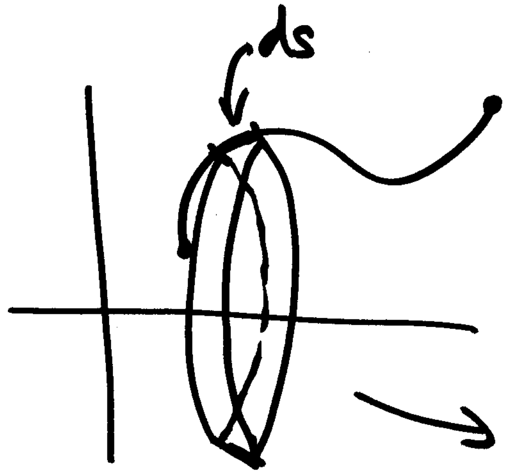
6.5 Surface areas of revolution

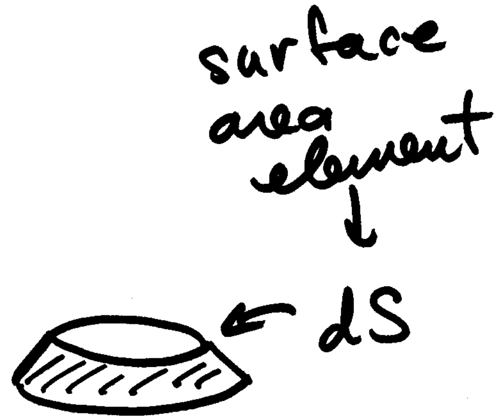
problem:



Rotating a curve about an axis gives a surface. Find area of the surface.

Idea:





"frustrum" of a cone

Need a formula for dS.

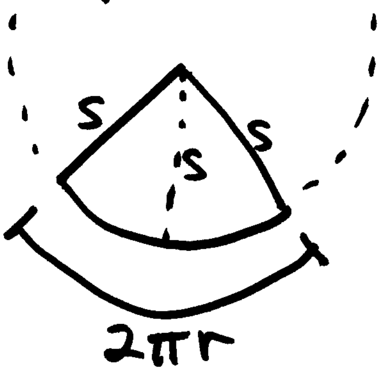
Cone:

r = radius

s = slant height

Want surface area A of cone.

split & spread out



$$A = \pi s^2 \cdot \frac{\cancel{2\pi} r}{\cancel{2\pi} s} = \pi r s$$

Frustrum:

want area of frustrum, A.

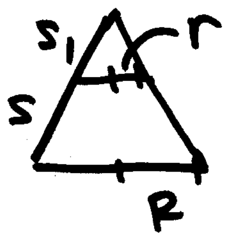
$$A = \pi R(s+s_1) - \pi r s_1$$

$$= \pi\big(\underbrace{(R-r)s_1}_{rs} + Rs\big)$$

$$= \pi(R+r)s$$

$$= 2\pi \underbrace{\left(\frac{R+r}{2}\right)}_{\text{avg radius}} \underset{\uparrow \text{ slant height.}}{s}$$

Look at similar Δ:

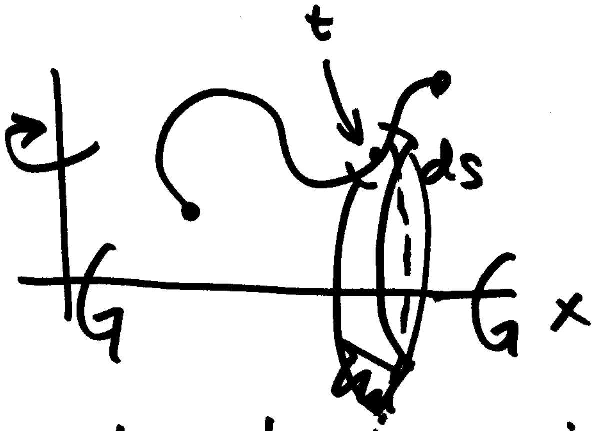


$$\frac{R}{r} = \frac{s+s_1}{s_1}$$

$$r(s+s_1) = Rs_1$$

$$rs = (R-r)s_1$$

Back to original problem.

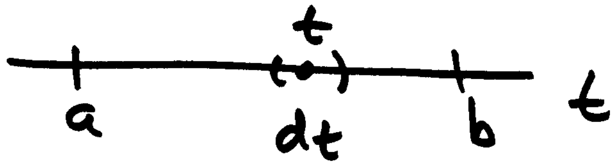


Rotate about x-axis

<u>e.g.</u> $x = x(t)$

$y = y(t)$

$a \le t \le b.$



$$dS = 2\pi\,(\text{radius})\,(\text{slant height})$$

$$= 2\pi\, y(t)\, ds$$

$$= 2\pi\, y(t) \left(\left[\frac{dx}{dt}\right]^2 + \left[\frac{dy}{dt}\right]^2 \right)^{1/2} dt$$

$$ds = (dx^2 + dy^2)^{1/2} = \left(\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 \right)^{1/2} dt$$

$$S = \int_a^b dS = \int_a^b 2\pi\, y(t) \left(\left[\frac{dx}{dt}\right]^2 + \left[\frac{dy}{dt}\right]^2 \right)^{1/2} dt$$

If we rotated curve about y-axis then

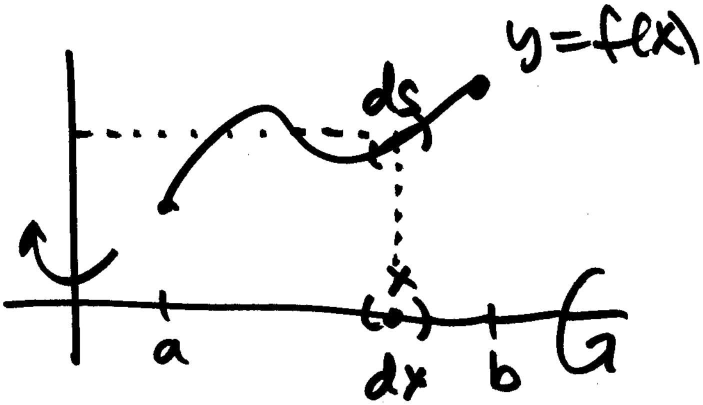
$$dS = 2\pi\, x(t)\, ds$$

$$= 2\pi\, x(t) \left(\left[\frac{dx}{dt}\right]^2 + \left[\frac{dy}{dt}\right]^2 \right)^{1/2} dt$$

$$S = \int_a^b dS = \int_a^b 2\pi x(t)\left(\left[\frac{dx}{dt}\right]^2 + \left[\frac{dy}{dt}\right]^2\right)^{1/2} dt$$

If curve given as $y = f(x)$ then

About xaxis :



$$ds = (dx^2 + dy^2)^{1/2}$$

$$= \left(dx^2\left(1 + \left(\frac{dy}{dx}\right)^2\right)\right)^{1/2}$$

$$= \left(1 + \left(\frac{dy}{dx}\right)^2\right)^{1/2} dx$$

$$dS = 2\pi f(x)\, ds = 2\pi f(x)\left(1 + f'(x)^2\right)^{1/2} dx$$

$$S = \int_a^b dS = \int_a^b 2\pi f(x)\left(1 + f'(x)^2\right)^{1/2} dx$$

About y-axis:

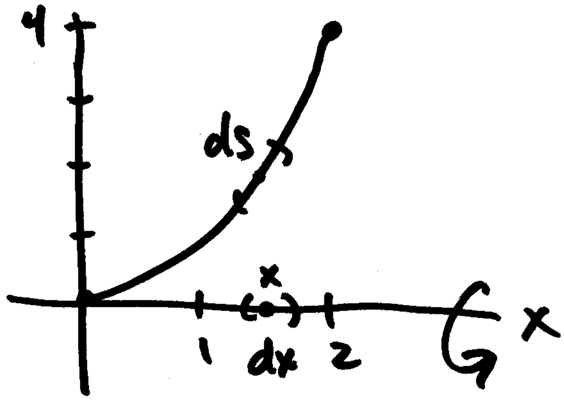
$$dS = 2\pi x\, ds = 2\pi x\left(1 + f'(x)^2\right)^{1/2} dx$$

$$S = \int_a^b dS = \int_a^b 2\pi x\left(1 + f'(x)^2\right)^{1/2} dx$$

eg # $\cancel{\ }$ p474

$y = x^2$, $0 \le x \le 2$

x-axis



$$dS = 2\pi x^2\, ds$$

$$= 2\pi x^2 (1+4x^2)^{1/2} dx$$

$$ds = (dx^2 + dy^2)^{1/2}$$

$$= \left(1 + \left[\frac{dy}{dx}\right]^2\right)^{1/2} dx$$

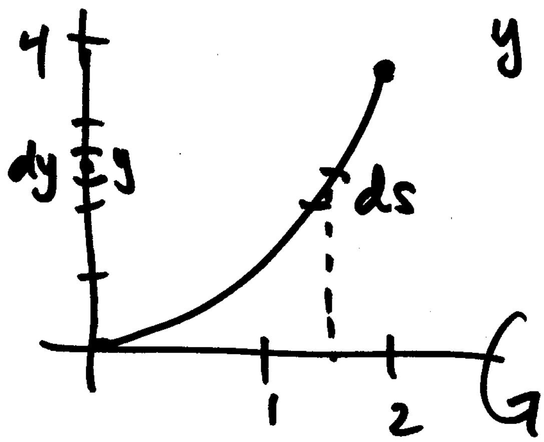
$$\frac{dy}{dx} = 2x$$

$$= \left(1 + (2x)^2\right)^{1/2} dx$$

$$= (1+4x^2)^{1/2} dx$$

$$S = \int_0^2 dS = \int_0^2 2\pi x^2 (1+4x^2)^{1/2} dx$$

Another approach to same problem



$y = x^2$

$$dS = 2\pi y \, ds$$
$$= 2\pi y \left(\frac{1}{4y} + 1\right)^{1/2} dy$$

$$ds = (dx^2 + dy^2)^{1/2}$$
$$= \left(\left[\frac{dx}{dy}\right]^2 + 1\right)^{1/2} dy$$
$$= \left(\left(\frac{1}{2} y^{-1/2}\right)^2 + 1\right)^{1/2} dy$$
$$= \left(\frac{1}{4y} + 1\right)^{1/2} dy$$

$$y = x^2$$
$$x = y^{1/2}$$
$$\frac{dx}{dy} = \frac{1}{2} y^{-1/2}$$

$$S = \int_0^4 dS = \int_0^4 2\pi y \left(\frac{1}{4y} + 1\right)^{1/2} dy.$$

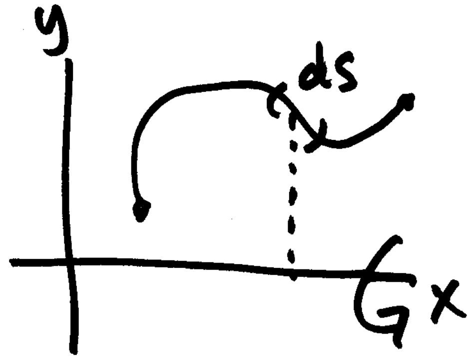
eg #36 (Take $a=1$)

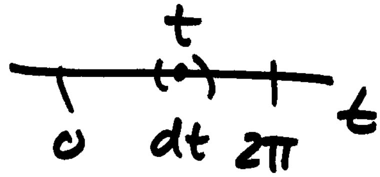
$x = t - \sin(t)$

$y = 1 - \cos(t)$

$0 \le t \le 2\pi$

rotate around x-axis





$dS = 2\pi\, y(t)\, \underline{ds}$

$= 2\pi(1-\cos(t)) \cdot (2-2\cos(t))^{1/2}\, dt$

$= \pi(2-2\cos(t)) \cdot (2-2\cos(t))^{1/2}\, dt$

$= \pi(2-2\cos(t))^{3/2}\, dt$

$$S = \int_0^{2\pi} \pi(2-2\cos(t))^{3/2}\, dt$$

$ds = (dx^2 + dy^2)^{1/2}$

$= \left(\left[\frac{dx}{dt}\right]^2 + \left[\frac{dy}{dt}\right]^2 \right)^{1/2} dt$

$\frac{dx}{dt} = 1 - \cos(t)$

$\frac{dy}{dt} = \sin(t)$

$= \left((1-\cos t)^2 + \sin^2 t\right)^{1/2} dt$

$= (1 - 2\cos t + \underbrace{\cos^2 t + \sin^2 t}_{1})^{1/2} dt$

$= (2 - 2\cos t)^{1/2} dt$