MATH 114 – 27 JULY 2007 – EXAM 4

Answer each of the following questions. Show all work, as partial credit may be given.

1. (5 pts. each) Consider the series $\sum_{n=1}^{\infty}\left(\frac{1}{2n+1}-\frac{1}{2n+3}\right)$.

   (a) Write out the first three terms of the *sequence of terms* for this series.

   (b) Write out the first three terms of the *sequence of partial sums* for this series.

   (c) Find a formula for s_n, the n^{th} partial sum of the series. (Hint: This is a telescoping series.)

   (d) Find the sum of the series.

2. (5 pts. each) Determine whether or not each of the following series converges or diverges. If it diverges, explain why it diverges. If it converges, find its sum.

   (a) $\sum_{n=0}^{\infty}(-1)^n\frac{5}{3^n}$

   (b) $\sum_{n=1}^{\infty}2^{1/n}$

   (c) $\sum_{n=4}^{\infty}\frac{2}{\sqrt{n}}$

3. (5 pts.) Express the repeating decimal $x = 0.212121212\cdots$ as the ratio of two integers.

4. (10 pts. each) Use the direct comparison or limit comparison test to determine if the following series converge. Justify your answer.

   (a) $\sum_{n=1}^{\infty}\frac{n}{\sqrt{n^3+1}}$

   (b) $\sum_{n=1}^{\infty}\frac{1}{1+3^n}$

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5. (10 pts. each) Use the ratio or root test to determine if the following series converge. Justify your answer.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{10^n}{n!}$

(b) $\displaystyle\sum_{n=3}^{\infty} \frac{n^3\, 2^n}{3^n}$

6. (10 pts.) Determine whether each of the following alternating series converges absolutely, converges conditionally, or diverges. Justify your answer.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3}{n^{2/5}}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-3)^n}{5^n}$