

MATH 114 – 20 JULY 2007 – EXAM 3

Answer each of the following questions. Show all work, as partial credit may be given.

1. (10 pts. each) Use an appropriate trigonometric substitution to convert each of the integrals below into an equivalent definite or indefinite trigonometric integral. DO NOT EVALUATE.

(a)  $\int_0^1 (1 - x^2)^{1/2} dx$

(b)  $\int \frac{x^2}{(x^2 - 1)^{3/2}} dx$

2. (10 pts. each) Use the integral table given below to evaluate the following integrals.

(a)  $\int \frac{x}{(2x + 1)^{3/2}} dx$

(b)  $\int \frac{dx}{(4 + x^2)^2}$

$$\begin{aligned}
 1. \int x(ax + b)^n dx &= \frac{(ax + b)^{n+1}}{a^2} \left[ \frac{ax + b}{n + 2} - \frac{b}{n + 1} \right] + C, n \neq -1, -2. \\
 2. \int \sqrt{a^2 + x^2} dx &= \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \ln(x + \sqrt{a^2 + x^2}) + C \\
 3. \int \frac{dx}{(a^2 + x^2)^2} dx &= \frac{x}{2a^2(a^2 + x^2)} + \frac{1}{2a^3} \tan^{-1} \frac{x}{a} + C \\
 4. \int \frac{dx}{(a^2 - x^2)^2} dx &= \frac{x}{2a^2(a^2 - x^2)} + \frac{1}{4a^3} \ln \left| \frac{x + a}{x - a} \right| + C
 \end{aligned}$$

3. (10 pts. each) Evaluate each of the following improper integrals.

(a)  $\int_2^\infty x e^{-x} dx$

(b)  $\int_0^1 \frac{dx}{(1 - x)^{1/2}}$

4. (10 pts. each) Use the limit comparison test to determine whether or not each of the following integrals converges. DO NOT EVALUATE.

(a)  $\int_1^{\infty} \frac{x+1}{x(x^2+1)} dx$

(b)  $\int_1^{\infty} \frac{y+4}{y(y+1)} dy$

5. (10 pts.) Consider the cardioid given by  $r = 1 + \cos(\theta)$ . Set up but DO NOT EVALUATE definite integrals giving the following.

(a) The area enclosed by the cardioid.

(b) The length of the cardioid.