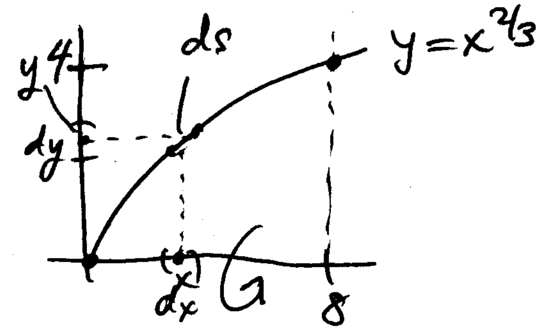
MATH 114 - EXAM 2 - SOLUTIONS

1. (a)



$$dS = 2\pi(\text{radius})(\text{slant height})$$
$$= 2\pi x^{2/3}\, ds$$
$$= 2\pi x^{2/3}\left(1+\tfrac{4}{9}x^{-2/3}\right)^{1/2} dx$$

$$ds = \left(1+\left(\frac{dy}{dx}\right)^2\right)^{1/2} dx$$
$$\frac{dy}{dx} = \frac{2}{3}x^{-1/3}$$
$$\therefore ds = \left(1+\tfrac{4}{9}x^{-2/3}\right)^{1/2} dx$$

$$S = \int_0^8 2\pi x^{2/3}\left(1+\frac{4}{9x^{2/3}}\right)^{1/2} dx \;//$$

(b) $dS = 2\pi y\, ds$

$$= 2\pi y\left(\tfrac{9}{4}y+1\right)^{1/2} dy$$

$$S = \int_0^4 2\pi y\left(\tfrac{9}{4}y+1\right)^{1/2} dy \;//$$

$$ds = \left(\left(\frac{dx}{dy}\right)^2+1\right)^{1/2} dy$$
$$y = x^{2/3} \qquad x = y^{3/2}$$
$$\frac{dx}{dy} = \frac{3}{2}y^{1/2}$$
$$ds = \left(\tfrac{9}{4}y+1\right)^{1/2} dy$$

2. $y(t) = y_0 e^{kt}$ $\qquad y_0 = 50000$

$$y(2) = y_0 e^{2k} = 50000\, e^{2k} = 62500$$

$$\therefore\ e^{2k} = \frac{62500}{50000} = 1.25$$

$$2k = \ln(1.25)$$

$$k = \frac{\ln(1.25)}{2} \approx .112$$

$$\therefore\ y(t) = 50000\, e^{.112t} \quad \text{or} \quad y(t) = 50000\, e^{\frac{\ln(5/4)}{2} t}$$

3. (a) $\displaystyle\int_0^1 x(x+1)^{1/2}\,dx$ $\qquad u = x+1$, $du = dx$, $x = u-1$; $x=0$, $u=1$; $x=1$, $u=2$

$$= \int_1^2 (u-1)u^{1/2}\,du$$

$$= \int_1^2 u^{3/2} - u^{1/2}\,du = \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\Big|_1^2$$

$$= \frac{2}{5}(2)^{5/2} - \frac{2}{3}(2)^{3/2} - \left(\frac{2}{5} - \frac{2}{3}\right)$$

$$= \frac{2}{5}2^{5/2} - \frac{2}{3}2^{3/2} + \frac{4}{15} \ //$$

$$= \frac{8}{5}\sqrt{2} - \frac{4}{3}\sqrt{2} + \frac{4}{15}$$

$$= \frac{24-20}{15}\sqrt{2} + \frac{4}{15} = \frac{4}{15}\sqrt{2} + \frac{4}{15}$$

(b) $2\int \frac{\cos(\sqrt{x})}{2\sqrt{x}} \cdot dx$ $\quad u=\sqrt{x}$, $\quad du = \frac{1}{2\sqrt{x}}\,dx$

$= 2\int \cos(u)\,du$

$= 2\sin(u) + C = 2\sin(\sqrt{x}) + C$ //

4. (a) $\int_1^4 x^2 \ln(x)\,dx$ $\quad u=\ln(x) \quad dv = x^2\,dx$, $\quad du = \frac{1}{x}\,dx \quad v=\frac{1}{3}x^3$

$= \frac{1}{3}x^3\ln(x)\Big|_1^4 - \frac{1}{3}\int_1^4 x^3\cdot\frac{1}{x}\,dx$

$= \frac{1}{3}(4)^3\ln(4) - \frac{1}{3}(1)^3\ln(1) - \frac{1}{3}\left(\int_1^4 x^2\,dx\right)$

$= \frac{64}{3}\ln(4) - \frac{1}{3}\cdot\frac{1}{3}x^3\Big|_1^4$

$= \frac{64}{3}\ln(4) - \frac{1}{9}(4^3-1^3) = \frac{64}{3}\ln(4) - 7$ //

(b) $\int \theta^2\sin(2\theta)\,d\theta$ $\quad \boxed{u=\theta^2 \quad dv=\sin(2\theta)\,d\theta \quad du = 2\theta\,d\theta \quad v=-\frac{1}{2}\cos(2\theta)}$

$= -\frac{1}{2}\theta^2\cos(2\theta) + \int \theta\cos(2\theta)\,d\theta$ $\quad u=\theta \quad dv=\cos(2\theta)\,d\theta$, $\quad du=d\theta \quad v=\frac{1}{2}\sin(2\theta)$

$= \frac{1}{2}\theta\sin(2\theta) - \frac{1}{2}\int \sin(2\theta)\,d\theta$

$= \frac{1}{2}\theta\sin(2\theta) + \frac{1}{4}\cos(2\theta) + C$

$= -\frac{1}{2}\theta^2\cos(2\theta) + \frac{1}{2}\theta\sin(2\theta) + \frac{1}{4}\cos(2\theta) + C$ //

5 (a) $\dfrac{x+1}{x(x^2+1)} = \dfrac{A}{x} + \dfrac{Bx+C}{x^2+1}$

$x+1 = A(x^2+1) + (Bx+C)x$

$\underline{x=0}$

$1 = A$ $\boxed{A=1}$

Derivative:

$1 = 2Ax + 2Bx + C$ $\underline{x=0}$

$1 = 0 + 0 + C$ $\boxed{C=1}$

Derivative:

$0 = 2A + 2B = 2 + 2B$ $\boxed{B=-1}$

$\dfrac{x+1}{x(x^2+1)} = \dfrac{1}{x} + \dfrac{-x+1}{x^2+1}$ //

(b) $\int \frac{y+4}{y(y+1)}\,dy$

$$\frac{y+4}{y(y+1)} = \frac{A}{y} + \frac{B}{y+1} = \frac{4}{y} - \frac{3}{y+1}$$

$y+4 = A(y+1) + By$ $\qquad \underline{y=0}$

$4 = A \qquad \boxed{A=4}$

$\underline{y=-1}$

$3 = -B \qquad \boxed{B=-3}$

$$= \int \frac{4}{y}\,dy - \int \frac{3}{y+1}\,dy$$

$$= 4\ln(y) - 3\ln(y+1) + C \;//$$

6. $\int_0^{\pi/2} \sin^4(x)\cos^3(x)\,dx$

$= \int_0^{\pi/2} \sin^4(x)\cos^2(x)\cos(x)\,dx$

$= \int_0^{\pi/2} \sin^4(x)(1-\sin^2(x))\cos(x)\,dx$

$u = \sin(x)$

$du = \cos(x)\,dx$

$x = 0 \quad u = 0$

$x = \frac{\pi}{2} \quad u = 1$

$= \int_0^1 u^4(1-u^2)\,du$

$= \int_0^1 u^4 - u^6\,du$

$= \frac{1}{5}u^5 - \frac{1}{7}u^7\Big|_0^1 = \frac{1}{5} - \frac{1}{7} = \frac{2}{35}$ //