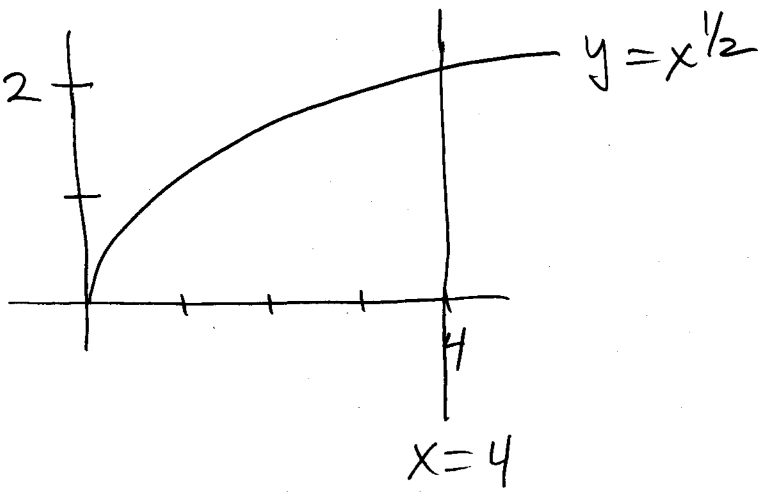
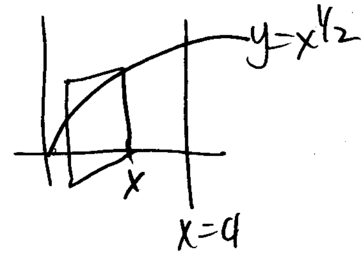
MATH 114 - EXAM 1 - SOLUTIONS

1.



(a) $A(x)$ = area of cross-section at x

$$= (x^{1/2})^2 = x$$

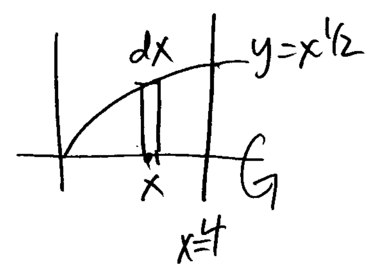


$$dV = A(x)\,dx = x\,dx$$

$$V = \int_0^4 x\,dx = \tfrac{1}{2}x^2\Big|_0^4 = 8$$

(b) $dV = \pi(\text{radius})^2\,dx$

$$= \pi (x^{1/2})^2\,dx = \pi x\,dx$$

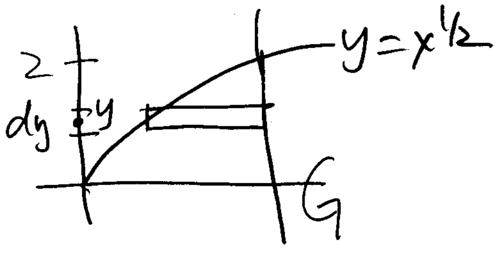


$$V = \int_0^4 \pi x\,dx = 8\pi$$

(c) $dV = 2\pi(\text{radius})(\text{height})(\text{thickness})$

$= 2\pi y(4-y^2)\,dy$

$V = \int_0^2 2\pi y(4-y^2)\,dy$



$= 2\pi\int_0^2 (4y-y^3)\,dy$

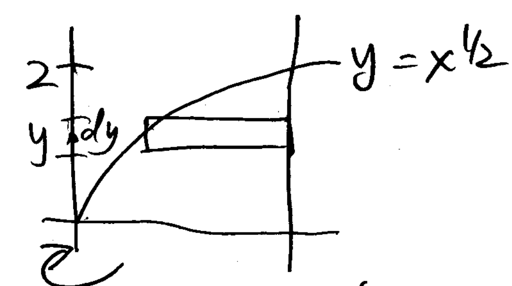
$x=y^2$

$= 2\pi\left(2y^2-\frac{1}{4}y^4\Big|_0^2\right)$

$= 2\pi(8-4-0) = 8\pi$ //

(d) $dV = \pi\left[\begin{pmatrix}\text{outer}\\\text{radius}\end{pmatrix}^2 - \begin{pmatrix}\text{inner}\\\text{radius}\end{pmatrix}^2\right]dy$

$= \pi(4^2 - (y^2)^2)\,dy$



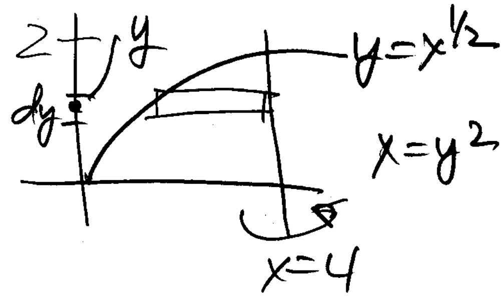
$= \pi(16-y^4)\,dy$

$V = \int_0^2 \pi(16-y^4)\,dy$

$= \pi\left(16y - \frac{1}{5}y^5\Big|_0^2\right)$

$= \pi\left(32 - \frac{32}{5} - 0\right) = \frac{128\pi}{5}$ //

(e) Disks:



$$dV = \pi(4-y^2)^2\,dy$$

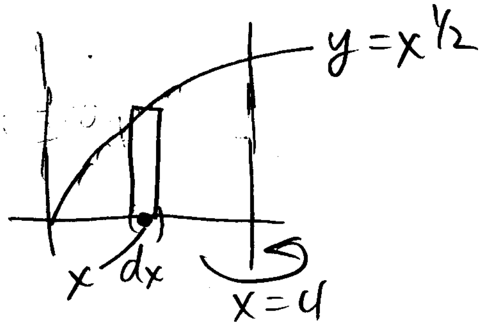
$$V = \int_0^2 \pi(4-y^2)^2\,dy = \pi\int_0^2 (16-8y^2+y^4)\,dy$$

$$= \pi\left(16y - \frac{8}{3}y^3 + \frac{1}{5}y^5 \Big|_0^2\right)$$

$$= \pi\left(32 - \frac{64}{3} + \frac{32}{5} - 0\right)$$

$$= \pi\left(\frac{480}{15} - \frac{320}{15} + \frac{96}{15}\right) = \frac{256\pi}{15} \,//$$

Shells:



$$dV = 2\pi(4-x)(x^{1/2})\,dx$$

$$V = \int_0^4 2\pi x^{1/2}(4-x)\,dx$$

$$= 2\pi\int_0^4 4x^{1/2} - x^{3/2}\,dx$$

$$= 2\pi\left(\frac{8}{3}x^{3/2} - \frac{2}{5}x^{5/2}\Big|_0^4\right)$$

$$= 2\pi\left(\frac{8}{3}(4)^{3/2} - \frac{2}{5}(4)^{5/2} - 0\right)$$

$$= 2\pi\left(\frac{64}{3} - \frac{64}{5}\right) = 128\pi\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{256\pi}{15} \,//$$

2. (a) $y = x^{3/2}$ $\quad \frac{dy}{dx} = \frac{3}{2}x^{1/2}$

$$ds = \left(1+\left(\frac{dy}{dx}\right)^2\right)^{1/2}dx = \left(1+\left(\frac{3}{2}x^{1/2}\right)^2\right)^{1/2}dx$$

$$= \left(1+\frac{9}{4}x\right)^{1/2}dx$$

$$L = \int_0^4 ds = \int_0^4 \left(1+\frac{9}{4}x\right)^{1/2}dx$$

$u = 1+\frac{9}{4}x$

$du = \frac{9}{4}dx$

$x=0 \quad u=1$

$x=4 \quad u=10$

$$= \frac{4}{9}\int_0^4 \left(1+\frac{9}{4}x\right)^{1/2}\cdot\left(\frac{9}{4}dx\right)$$

$$= \frac{4}{9}\int_1^{10} u^{1/2}du = \frac{4}{9}\cdot\frac{2}{3}u^{3/2}\Big|_1^{10}$$

$$= \frac{8}{27}\left(10^{3/2}-1\right) /\!/$$

(b) $x = \frac{2}{3}t^{3/2}$ $\quad y = \frac{1}{2}t^2 - \frac{1}{4}t$

$\frac{dx}{dt} = t^{1/2}$ $\quad \frac{dy}{dt} = t - \frac{1}{4}$

$$ds = \left(\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right)^{1/2}dt$$

$$= \left(t + \left(t^2 - \frac{1}{2}t + \frac{1}{16}\right)\right)^{1/2}dt$$

$$= \left(t^2 + \frac{1}{2}t + \frac{1}{16}\right)^{1/2}dt = \left(\left(t+\frac{1}{4}\right)^2\right)^{1/2}dt = \left(t+\frac{1}{4}\right)dt$$

$$L = \int_1^8 \left(t + \frac{1}{4}\right) dt = \frac{1}{2}t^2 + \frac{1}{4}t \Big|_1^8$$

$$= 32 + 2 - \frac{1}{2} - \frac{1}{4} = 34 - \frac{3}{4} = 33\frac{1}{4} = \frac{133}{4} \,//$$

3. $M_0 = \int_0^5 x(25-x^2)^{1/2}\,dx$ $\qquad u = 25 - x^2$, $du = -2x\,dx$

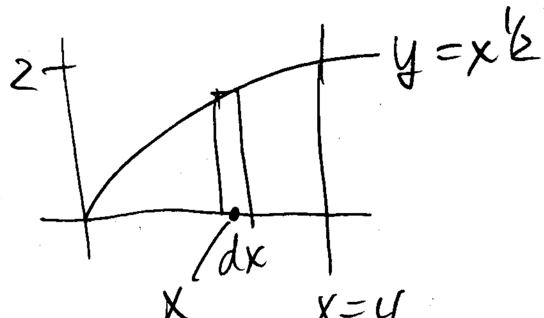
$$= -\frac{1}{2}\int_0^5 (25-x^2)^{1/2}(-2x)\,dx$$ $\qquad x=0 \;\; u=25$, $x=5 \;\; u=0$

$$= -\frac{1}{2}\int_{25}^{0} u^{1/2}\,du = \frac{1}{2}\int_0^{25} u^{1/2}\,du$$

$$= \frac{1}{2}\cdot\frac{2}{3}u^{3/2}\Big|_0^{25} = \frac{1}{3}(25)^{3/2} - 0 = \frac{125}{3} \,//.$$

$$\bar{x} = \frac{\frac{125}{3}}{\frac{25\pi}{4}} = \frac{\overset{5}{\cancel{125}}}{3}\cdot\frac{4}{\cancel{25}\pi} = \frac{20}{3\pi} \approx 2.12 \,//$$

4. (a)

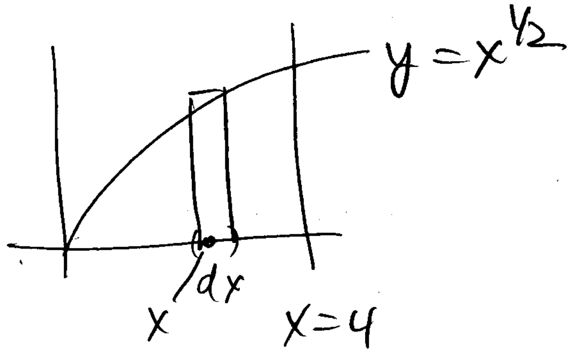


$$M_x = \int_0^4 \frac{1}{2}x^{1/2}\,dm$$

$$= \int_0^4 \frac{1}{2}x^{1/2}\cdot x^{1/2}\,dx$$

$$= \int_0^4 \frac{1}{2}x\,dx = 4 \,//$$

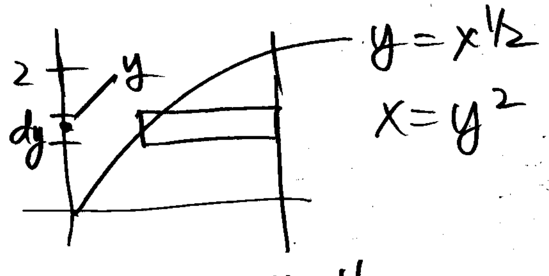
(b)



$$M_y = \int_0^4 x \, dm = \int_0^4 x \cdot x^{1/2} \, dx = \int_0^4 x^{3/2} \, dx$$

$$= \frac{2}{5} x^{5/2} \Big|_0^4 = \frac{2}{5}\left(4^{5/2} - 0\right) = \frac{64}{5} \; //.$$

Alternate solution for M_x:



$$M_x = \int_0^2 y \, dm$$

$$= \int_0^2 y(4 - y^2) \, dy$$

$$= \int_0^2 4y - y^3 \, dy$$

$$= \left(2y^2 - \frac{1}{4}y^4\right)\Big|_0^2$$

$$= 8 - 4 - 0 = 4 \; //.$$