MATH 114 – 9 JULY 2007 – EXAM 1

Answer each of the following questions. Show all work, as partial credit may be given.

1. (10 pts. each) Consider the region in the first quadrant bounded by the x–axis, the curve $y = x^{1/2}$, and the line $x = 4$.

(a) Find the volume of the solid whose base is the region given above and whose cross-sections perpendicular to the x-axis are squares with base in the x–y plane.

(b) Use the method of disks to find the volume of the solid obtained by rotating the above region about the x–axis.

(c) Using the method of cylindrical shells, find the volume of the solid obtained by rotating the above region about the x–axis.

(d) Use the method of washers to set up an integral giving the volume of the solid obtained by rotating the above region about the line y–axis. DO NOT EVALUATE.

(e) Use any method you like to set up an integral giving the volume of the solid obtained by rotating the above region about the line $x = 4$. DO NOT EVALUATE.

2. (10 pts. each)

(a) Find the length of the curve given by $y = x^{3/2}$, $0 \leq x \leq 4$.

(b) Set up an integral giving the length of the curve given by the parametric equations $x = (2/3)t^{3/2}$, $y = (1/2)t^2 - (1/4)t$, $1 \leq t \leq 8$. DO NOT EVALUATE.

3. (10 pts.) Suppose that the density function of a thin rod lying along the interval $0 \leq x \leq 5$ is given by $\delta(x) = (25 - x^2)^{1/2}$. Find M_0, the rod's moment about the origin. Given that the mass of the rod is $M = 25\pi/4$, find the center of mass of the rod.

4. (10 pts. each) Suppose that a thin plate covers the region given in Problem 1, and has constant density $\delta = 1$.

(a) Find M_x, the moment of the region about the x–axis.

(b) Find M_y, the moment of the region about the y–axis.