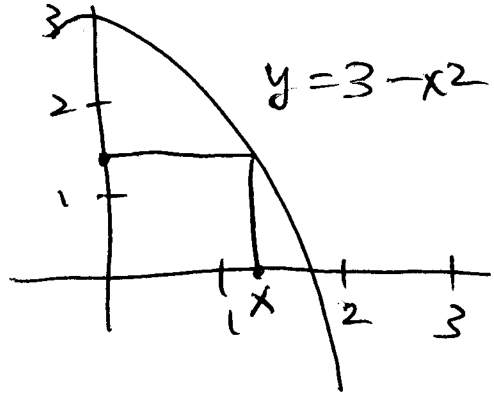
MATH 113 – EXAM 4 – SOLUTIONS

1.



$A = x(3-x^2) = 3x - x^3$

$0 \le x \le \sqrt{3}$

$A' = 3 - 3x^2$

$3 - 3x^2 = 0$

$3(1-x^2) = 0$

$\underline{\underline{x=1}}, \ \cancel{x=-1}$

$A(0) = A(\sqrt{3}) = 0$

$A(1) = 3(1) - (1)^3 = 2$

$\therefore$ abs maximum

Area is maximized when $x = 1$ //, $y = 3 - (1)^2 = 2$ //

2. (a) $\displaystyle\lim_{t\to 3} \frac{(t+1)^{1/2} - 2}{t-3}$ (Form $\frac{0}{0}$)

$\displaystyle\overset{L'H}{=} \lim_{t\to 3} \frac{\frac{1}{2}(t+1)^{-1/2}}{1} = \frac{1}{2}(3+1)^{-1/2} = \frac{1}{4}$ //

(b) $\displaystyle\lim_{x\to 0^+} x^{1/2} \ln(x)$ (Form $0\cdot\infty$)

$\displaystyle= \lim_{x\to 0^+} \frac{\ln(x)}{x^{-1/2}}$ (Form $\frac{\infty}{\infty}$)

$\displaystyle\overset{L'H}{=} \lim_{x\to 0^+} \frac{1/x}{-\frac{1}{2}x^{-3/2}} = \lim_{x\to 0^+} -2 \cdot \frac{x^{3/2}}{x} = \lim_{x\to 0^+} -2 \cdot x^{1/2} = 0$ //

(c) $\lim_{x \to \frac{\pi}{2}} \dfrac{1-\sin(x)}{1+\cos(2x)}$ $\left(\text{Form } \frac{0}{0}\right)$

$\stackrel{L'H}{=} \lim_{x \to \frac{\pi}{2}} \dfrac{+\cos(x)}{+2\sin(2x)}$ $\left(\text{Form } \frac{-\cos \pi/2}{-2\sin \pi} = \frac{0}{0}\right)$

$\stackrel{L'H}{=} \lim_{x \to \frac{\pi}{2}} \dfrac{-\sin(x)}{4\cos(2x)} = \dfrac{-\sin(\pi/2)}{4\cos(\pi)} = \dfrac{1}{4}$ //.

3. $f(x) = x^5 - x - 1$

$f'(x) = 5x^4 - 1$

$$\boxed{x_{n+1} = x_n - \frac{x_n^5 - x_n - 1}{5x_n^4 - 1}}$$

$x_0 = 1$

$x_1 = 1 - \dfrac{1-1-1}{5-1} = \dfrac{5}{4}$ //

$x_2 = 1.25 - \dfrac{(1.25)^5 - (1.25) - 1}{5(1.25)^4 - 1} = 1.17846$ //

4. (a) $\int_1^4 \left(x^{3/2} + 3x^{-1/2}\right) dx$

$$= \frac{2}{5}x^{5/2} + 6x^{1/2} \Big|_1^4$$

$$= \frac{2}{5}(4)^{5/2} + 6(4)^{1/2} - \left(\frac{2}{5}(1)^{5/2} + 6(1)^{1/2}\right)$$

$$= \frac{2}{5}\cdot 32 + 6\cdot 2 - \frac{2}{5} - 6$$

$$= \frac{64}{5} + 12 - \frac{2}{5} - 6 = \frac{62}{5} + \frac{30}{5} = 92/5 \;//$$

(b) $\int \left(\frac{2}{x} + e^{3x}\right) dx = 2\ln(x) + \frac{1}{3}e^{3x} + C \;//$

(c) $\cancel{\int_0^{\pi/4} \cos(3x)\,dx = \frac{1}{3}\sin(3x)\Big|_0^{\pi/4}}$

$$\int_0^{\pi/2} \cos\left(\tfrac{1}{3}x\right) dx = 3\sin\left(\tfrac{1}{3}x\right)\Big|_0^{\pi/2}$$

$$= 3\sin\left(\frac{\pi}{6}\right) - 3\sin(0)$$

$$= 3\cdot\frac{1}{2} = 3/2 \;//$$

5. $\displaystyle\int \frac{dx}{x(\ln x)^2}$ $\qquad u = \ln(x)$, $\quad du = \frac{1}{x}\,dx$

$$= \int \frac{1}{u^2}\,du = \int u^{-2}\,du = -u^{-1} + c$$

$$= -\frac{1}{\ln x} + c \quad //$$

6. (a) $\displaystyle\int \frac{dx}{(5x+8)^{1/2}}$ $\qquad u = 5x+8$, $\quad du = 5\,dx$

$$= \frac{1}{5}\int \frac{5\,dx}{(5x+8)^{1/2}} = \frac{1}{5}\int u^{-1/2}\,du$$

$$= \frac{2}{5}u^{1/2} + c = \frac{2}{5}(5x+8)^{1/2} + c \quad //$$

(b) $\displaystyle\int t^3(1+t^4)^3\,dt$ $\qquad u = 1+t^4$, $\quad du = 4t^3\,dt$

$$= \frac{1}{4}\int 4t^3(1+t^4)^3\,dt$$

$$= \frac{1}{4}\int u^3\,du = \frac{1}{16}u^4 + c$$

$$= \frac{1}{16}(1+t^4)^4 + c \quad //$$