MATH 113 – 15 JUNE 2007 – EXAM 4

Answer each of the following questions. Show all work, as partial credit may be given.

1. (10 pts.) A rectangle in the first quadrant has its base on the positive x–axis, one vertical side on the positive y–axis, and its upper right corner on the curve $y = 3 - x^2$. Find the dimensions of the rectangle with largest area.

2. Find the following limits using L'Hopital's rule.

(a) (5 pts.) $\lim_{t \to 3} \dfrac{(t+1)^{1/2} - 2}{t - 3}$

(b) (5 pts.) $\lim_{x \to 0^+} x^{1/2} \ln(x)$

(c) (10 pts.) $\lim_{x \to \pi/2} \dfrac{1 - \sin(x)}{1 + \cos(2x)}$

3. (10 pts.) Find the first two Newton iterates (that is, x_1 and x_2) for the equation $x^5 - x - 1 = 0$ when $x_0 = 1$. Write your answers correct to five decimal places.

4. (10 pts. each) Evaluate the following integrals.

(a) $\displaystyle\int_1^4 (x^{3/2} + 3x^{-1/2})\, dx$

(b) $\displaystyle\int \left(\frac{2}{x} + e^{3x}\right) dx$

(c) $\displaystyle\int_0^{\pi/2} \cos\left(\frac{1}{3}x\right) dx$

5. (10 pts.) Use the substitution $u = \ln(x)$ to reduce the integral $\displaystyle\int \frac{dx}{x(\ln x)^2}$ to an equivalent integral in the variable u, then evaluate the integral. Show all work.

6. (10 pts. each) Evaluate the following indefinite integrals by making an appropriate substitution.

(a) $\displaystyle\int \frac{dx}{(5x+8)^{1/2}}$

(b) $\displaystyle\int t^3(1+t^4)^3\, dt$