MATH 113 - EXAM 3 - SOLUTIONS

1. $y = \dfrac{(x+1)^{3/2}}{(x^2+1)^{5/4}}$

$\ln(y) = \frac{3}{2}\ln(x+1) - \frac{5}{4}\ln(x^2+1)$

$\frac{1}{y}\frac{dy}{dx} = \frac{3}{2(x+1)} - \frac{5}{\cancel{2}4}\cdot\frac{\cancel{2}x}{x^2+1} = \frac{3}{2(x+1)} - \frac{5x}{2(x^2+1)}$ //

2. $V = \frac{1}{3}\pi r^2 h$

$r = 10$
$h = 15$
$\frac{dr}{dt} = -2$

$\frac{dV}{dt} = \frac{1}{3}\pi\left(r^2\frac{dh}{dt} + 2rh\frac{dr}{dt}\right)$

$200\pi = \frac{1}{3}\pi\left(10^2\frac{dh}{dt} + 2(10)(15)(-2)\right)$

$= \frac{1}{3}\pi\left(100\frac{dh}{dt} - 600\right)$

$= \frac{100\pi}{3}\frac{dh}{dt} - 200\pi$

$\therefore \frac{dh}{dt} = \frac{(200\pi + 200\pi)(3)}{100\pi} = 12$ cm/min

3. (a) $f(x) = x(\ln x)^2$

$f'(x) = x \cdot 2(\ln x) \cdot \frac{1}{x} + (\ln x)^2 (1)$

$= 2\ln x + (\ln x)^2$ //

(b) $f(x) = \ln\left(\frac{1}{x\sqrt{x+1}}\right) = -\ln(x) - \frac{1}{2}\ln(x+1)$

$= -\frac{1}{x} - \frac{1}{2(x+1)}$ //

(c) $h(x) = \sin^{-1}(x^2-1)$

$h'(x) = \frac{1}{\sqrt{1-(x^2-1)^2}}(2x) = \frac{2x}{\sqrt{1-x^4+2x^2-1}}$

$= \frac{2x}{\sqrt{2x^2-x^4}}$ //

(d) $y = (x+2)^x$

$\ln y = x\ln(x+2)$

$\frac{1}{y}\frac{dy}{dx} = x \cdot \frac{1}{x+2} + \ln(x+2)$

$\frac{dy}{dx} = (x+2)^x\left(\frac{x}{x+2} + \ln(x+2)\right)$

4. $y = \tan^{-1}(x^2)$

$$\frac{dy}{dx} = \frac{1}{1+x^4} \cdot 2x = \frac{2x}{1+x^4} \,//$$

$$\frac{d^2y}{dx^2} = \frac{(1+x^4)(2) - (2x)(4x^3)}{(1+x^4)^2} = \frac{1+2x^4 - 8x^4}{(1+x^4)^2}$$

$$= \frac{1-6x^4}{(1+x^4)^2} \,//$$

5. $L(x) = f(a) + f'(a)(x-a)$

$f(x) = \ln(x^2-3)$ $\qquad$ $f(2) = \ln(1) = \underline{0}$

$f'(x) = \dfrac{2x}{x^2-3}$ $\qquad$ $f'(2) = \dfrac{4}{4-3} = 4$

$L(x) = 4(x-2) = 4x-8 \,//$

6. $y = 5x^4 - x^5$

(a) $dy = (20x^3 - 5x^4)\,dx$

(b) $x = 1 \quad dx = -.2$

$\Delta y \approx dy = (20(1)^3 - 5(1)^4)(-.2)$

$= 15(-.2) = -3$

$\Delta y = f(.8) - f(1)$

$= 5(.8)^4 - (.8)^5 - 5(1)^4 + (1)^5$

$= -2.27$

7. (a) $f(x) = x^4 - 8x^2 + 12$

$f'(x) = 4x^3 - 16x$

$4x^3 - 16x = 0$

$4x(x^2 - 4) = 0$

$x = 0,\ x = 2,\ x = -2$

$- \; 0 \; + \; 0 \; - \; 0 \; +$

$-2 \quad 0 \quad 2 \quad f'$

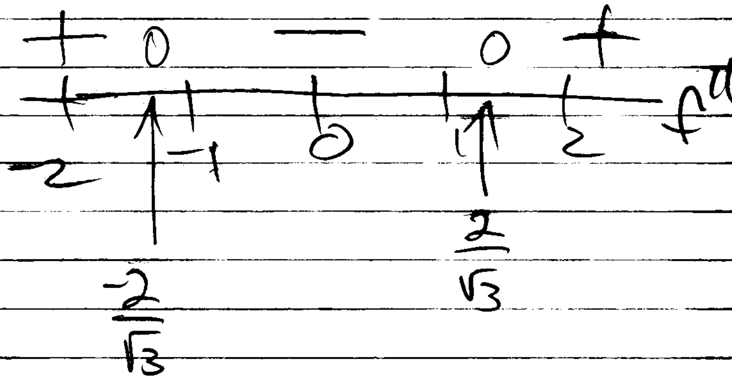
$f'(-3) < 0 \quad f'(1) < 0$

$f'(-1) > 0 \quad f'(3) > 0$

Local min at $x = 2, -2$

Local max at $x = 0$

(b) $f''(x) = 12x^2 - 16$



$12x^2 - 16 = 0$

$4(3x^2 - 4) = 0$

$x = \frac{2}{\sqrt{3}}$ $\quad x = -\frac{2}{\sqrt{3}}$

$f''(-2) > 0$

$f''(0) < 0$

$f''(2) > 0$

f(x) concave up on $(-\infty, -2/\sqrt{3}) \cup (2/\sqrt{3}, \infty)$ //

concave down on $\left(\frac{-2}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right)$ //

inflection points at $x = \pm \frac{2}{\sqrt{3}}$ //