MATH 113 – 8 JUNE 2007 – EXAM 3

Answer each of the following questions. Show all work, as partial credit may be given.

1. (10 pts.) Use logarithmic differentiation to find the derivative of $y = \dfrac{(x+1)^{3/2}}{(x^2+1)^{5/4}}$.

2. (10 pts.) Suppose that the volume V of a right circular cone is increasing at a rate of $200\pi\, cm^3/min$. If the radius r is *decreasing* at a rate of $2\, cm/min$ at what rate is the height h of the cone changing when $r = 10$ and $h = 15$? Be sure to put your answer in correct units. (Hint: The volume V of a right circular cone with radius r and height h is $V = (1/3)\,\pi r^2 h$).

3. (5 pts. each) Find the first derivative of each of the following functions.

   (a) $f(x) = x\,(\ln(x))^2$

   (b) $f(x) = \ln\left(\dfrac{1}{x\sqrt{x+1}}\right)$

   (c) $h(x) = \sin^{-1}(x^2 - 1)$

   (d) $y = (x+2)^x$ (Hint: Use logarithmic differentiation.)

4. (10 pts.) Find the first and second derivative of the function $y = \tan^{-1}(x^2)$.

5. (10 pts.) Find the linearization of the function $f(x) = \ln(x^2 - 3)$ at $x = 2$.

6. (10 pts. each) Let $y = 5x^4 - x^5$.

   (a) Find the differential dy.

   (b) Find the actual change in y, Δy, when x changes from 1 to .8, then use the differential you found in part (a) to *estimate* Δy when x changes from 1 to .8.

7. (10 pts. each) Let $f(x) = x^4 - 8x^2 + 12$

   (a) Find all critical points for $f(x)$ and identify each as a local maximum, local minimum, or neither. Show all work.

   (b) Find the intervals on which $f(x)$ is concave up and concave down, and the location of all inflection points of $f(x)$.