MATH 113 - EXAM 2 - SOLUTIONS

1. $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 + (x+h) - (x^2 + x)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{x} + h - \cancel{x^2} - \cancel{x}}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 + h}{h} = \lim_{h \to 0} \frac{\cancel{h}(2x + h + 1)}{\cancel{h}} = 2x+1 \,//$$

2. (a) $f(x) = x^2 + x - 8x^{-1}$

$f'(x) = 2x + 1 + 8x^{-2}$ //

(b) $y = (x + \frac{1}{x})(x+1)$

Product rule: $y' = (x + \frac{1}{x}) \frac{d}{dx}(x+1) + (x+1) \frac{d}{dx}(x + \frac{1}{x})$

$= (x + \frac{1}{x})(1) + (x+1)(1 - \frac{1}{x^2}) = x + \cancel{\frac{1}{x}} + x - \cancel{\frac{1}{x}} + 1 - \frac{1}{x^2}$

$= 2x + 1 - \frac{1}{x^2}$ //

Simplify: $y = (x + \frac{1}{x})(x+1) = x^2 + x + 1 + \frac{1}{x}$

$y' = 2x + 1 - \frac{1}{x^2}$ //

(c) $f(t) = \dfrac{2t+1}{1-t^2}$

$$f'(t) = \frac{(1-t^2)\frac{d}{dt}(2t+1) - (2t+1)\frac{d}{dt}(1-t^2)}{(1-t^2)^2}$$

$$= \frac{(1-t^2)(2) - (2t+1)(-2t)}{(1-t^2)^2}$$

$$= \frac{2-2t^2+4t^2+2t}{(1-t^2)^2}$$

$$= \frac{2t^2+2t+2}{(1-t^2)^2} \;//.$$

(d) $y = x^2 e^{-2x}$

$$y' = x^2 \frac{d}{dx} e^{-2x} + e^{-2x}\frac{d}{dx}(x^2)$$

$$= x^2(-2e^{-2x}) + e^{-2x}(2x)$$

$$= e^{-2x}(2x-2x^2).$$

$$= 2x(1-x)(e^{-2x}) //$$

(e) $f(x) = \frac{\cos(x)}{x}$

$$f'(x) = \frac{x \frac{d}{dx}\cos(x) - \cos(x)\frac{d}{dx}(x)}{x^2}$$

$$= \frac{-x\sin(x) - \cos(x)}{x^2} \ //$$

(f) $r = \tan^3(\theta)$

$$\frac{dr}{d\theta} = 3\tan^2(\theta)\frac{d}{d\theta}\tan(\theta)$$

$$= 3\tan^2(\theta)\sec^2(\theta) \ //$$

3. (a) $f(x) = \frac{2x^2+5}{x}$

simplify: $f(x) = 2x + 5x^{-1}$

$$f'(x) = 2 - 5x^{-2} = 2 - \frac{5}{x^2} \ //$$

$$f''(x) = 10x^{-3} = \frac{10}{x^3} \ //$$

Quotient rule:

$$f'(x) = \frac{x(4x)-(2x^2+5)(1)}{x^2} = \frac{4x^2-2x^2-5}{x^2} = \frac{2x^2-5}{x^2} \;//$$

$$f''(x) = \frac{x^2(4x)-(2x^2-5)(2x)}{x^4} = \frac{4x^3-4x^3+10x}{x^4} = \frac{10x}{x^4}$$

$$= \frac{10}{x^3} \;//$$

(b) $y = \sin(x^2)$

$y' = 2x\cos(x^2)$ //

$$y'' = 2x \frac{d}{dx}\cos(x^2) + \cos(x^2)\frac{d}{dx}(2x)$$

$$= 2x(-2x\sin(x^2)) + 2\cos(x^2)$$

$$= -4x^2\sin(x^2) + 2\cos(x^2) \;//$$

4. $s = t^3 - 12t^2 + 36t \quad 0 \le t \le 8$

(a) $v = s' = 3t^2 - 24t + 36$ //

$a = s'' = 6t - 24$ //

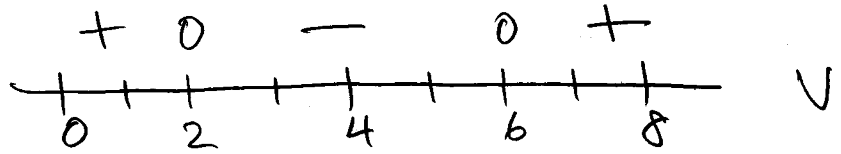
(b) $3t^2 - 24t + 36 = 0$

$3(t^2 - 8t + 12) = 0$

$3(t-2)(t-6) = 0$

Body at rest when $t=2$, $t=6$ sec. //

(c)



Body moves right : $(0,2) \cup (6,8)$ //

Body moves left : $(2,6)$ //

5. $y^3 - 3xy = x^3 - y \qquad (-2,-1)$ //

$$\frac{d}{dx}(y^3 - 3xy) = \frac{d}{dx}(x^3 - y)$$

$$3y^2\frac{dy}{dx} - 3x\frac{dy}{dx} - 3y = 3x^2 - \frac{dy}{dx}$$

If $x = -2,\ y = -1$:

$$3(-1)^2\frac{dy}{dx} - 3(-2)\frac{dy}{dx} - 3(-1) = 3(-2)^2 - \frac{dy}{dx}$$

$$3\frac{dy}{dx} + 6\frac{dy}{dx} + 3 = 12 - \frac{dy}{dx}$$

$$10\frac{dy}{dx} = 9 \qquad \left.\frac{dy}{dx}\right|_{(-2,-1)} = \frac{9}{10} \text{ //}$$

Eqn of Tangent line:

$$y + 1 = \frac{9}{10}(x+2)$$

$$y = \frac{9}{10}x + \frac{4}{5} \text{ //}$$