MATH 113 – 25 MAY 2007 – EXAM 1

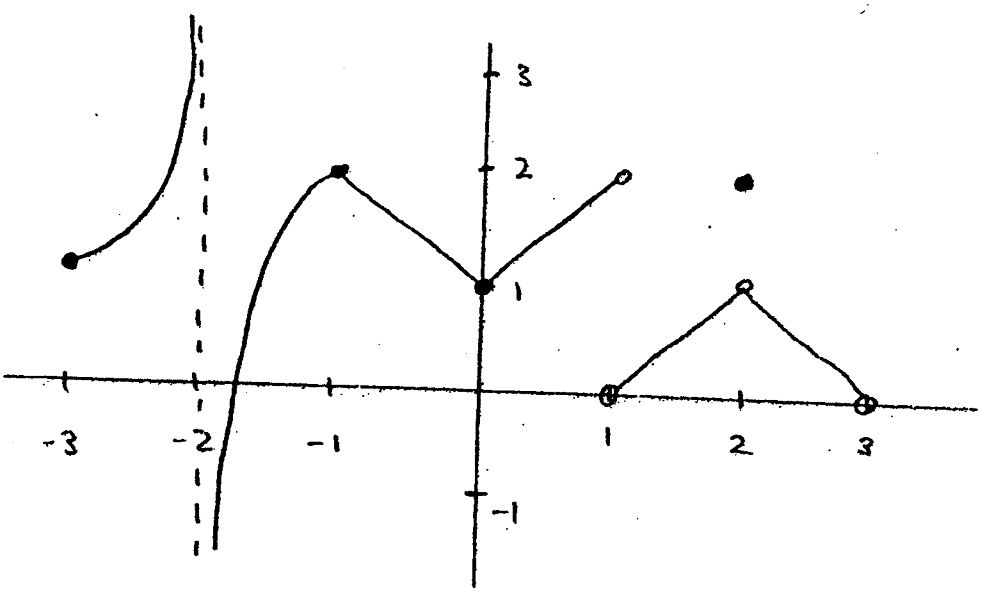
Answer each of the following questions. Show all work, as partial credit may be given.

1. (10 pts. each) Let $f(x) = x^2 + x$.

   (a) Find the *average rate of change* of $f(x)$ over the interval $[1, 2]$ and over the interval $[1, 1.5]$.

   (b) Find the *instantaneous rate of change* of $f(x)$ with respect to x at $x = 1$. (Hint: This must be done by evaluating an appropriate limit.)

2 . (5 pts. each) Consider the function $f(x)$ whose graph is sketched below and answer the following questions.



(a) Does $\lim_{x\to 2} f(x)$ exist? If so, find its value. If not, explain why it does not exist.

(b) Is $f(x)$ continuous at $x = 2$? Fully explain your answer.

(c) Does $\lim_{x\to 1} f(x)$ exist? If so, find its value. If not, explain why it does not exist.

(d) Does $\lim_{x\to 0^+} f(x)$ exist? If so, find its value. If not, explain why it does not exist. Do the same for $\lim_{x\to 0^-} f(x)$.

(e) Is $f(x)$ continuous at $x = -1$? Fully explain your answer.

(f) Find each of the infinite limits $\lim_{x\to -2^+} f(x)$ and $\lim_{x\to -2^-} f(x)$.

3. (10 pts. each) Evaluate the following limits.

(a) $\lim_{x\to 2} \dfrac{x+2}{x^2-3x-10}$

(b) $\lim_{x\to -2} \dfrac{x+2}{x^2-3x-10}$

(c) $\lim_{x\to \infty} \dfrac{x+2}{x^2-3x-10}$

(d) $\lim_{x\to 5+} \dfrac{x+2}{x^2-3x-10}$

4. (10 pts.) Find the slope of the tangent line to the graph of the function $f(x) = \sqrt{x+1}$ at $x = 0$ by calculating the limit of the difference quotient. Then find an equation for the line tangent to the graph there.

MATH 113 – EXAM 1 – SOLUTIONS

1. (a) avg. r.a.c. over $[1,2]$ $= \dfrac{f(2)-f(1)}{2-1} = \dfrac{6-2}{2-1} = 4$ //

avg r.a.c. over $[1,1.5]$ $= \dfrac{f(1.5)-f(1)}{1.5-1} = \dfrac{3.75-2}{.5} = 3.5$ //

(b) inst r.a.c. at $x=1$ $= \lim\limits_{h\to 0} \dfrac{f(1+h)-f(1)}{h}$

$$= \lim_{h\to 0} \frac{(1+h)^2+(1+h)-2}{h} = \lim_{h\to 0} \frac{\cancel{1}+2h+h^2+\cancel{1}+h-\cancel{2}}{h}$$

$$= \lim_{h\to 0} \frac{3h+h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(3+h)}{\cancel{h}} = \lim_{h\to 0} 3+h = 3 \text{ //}$$

2. (a) Yes, $\lim_{x \to 2} f(x) = 1$

(b) No since $\lim_{x \to 2} f(x) = 1$ but $f(2) = 2$,

so $\lim_{x \to 2} f(x) \neq f(2)$.

(c) $\lim_{x \to 1} f(x)$ does not exist because

$2 = \lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x) = 0$.

(d) Yes both exist and both equal 1.

(e) Yes because $\lim_{x \to -1} f(x) = 2 = f(-1)$.

(f) $\lim_{x \to -2^+} f(x) = -\infty$ $\lim_{x \to -2^-} f(x) = +\infty$.

3 (a) $\lim\limits_{x\to 2} \dfrac{x+2}{x^2-3x-10} = \dfrac{2+2}{2^2-3\cdot 2-10} = \dfrac{4}{-12} = \dfrac{-1}{3}$ //

(b) $\lim\limits_{x\to -2} \dfrac{x+2}{x^2-3x-10} = \lim\limits_{x\to -2} \dfrac{\cancel{x+2}}{(\cancel{x+2})(x-5)} =$

$= \lim\limits_{x\to -2} \dfrac{1}{x-5} = \dfrac{-1}{7}$ //.

(c) $\lim\limits_{x\to \infty} \dfrac{x+2}{x^2-3x-10} = \lim\limits_{x\to\infty} \dfrac{x}{x^2} = \lim\limits_{x\to\infty} \dfrac{1}{x} = 0$ //

(d) $\lim\limits_{x\to 5^+} \dfrac{x+2}{(x+2)(x-5)} = \lim\limits_{x\to 5^+} \dfrac{1}{x-5} = +\infty$

(since $x-5>0$ if $x>5$).

4. slope of tangent line at $x=0$ $= \lim_{h \to 0} \frac{f(0+h)-f(0)}{h}$

$$= \lim_{h \to 0} \frac{f(h)-f(0)}{h} = \lim_{h \to 0} \frac{\sqrt{h+1}-1}{h}$$

$$\boxed{\frac{\sqrt{h+1}-1}{h} \cdot \frac{\sqrt{h+1}+1}{\sqrt{h+1}+1} = \frac{\cancel{h}+\cancel{1}-\cancel{1}}{\cancel{h}(\sqrt{h+1}+1)} = \frac{1}{\sqrt{h+1}+1}}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{h+1}+1} = \frac{1}{2} \,//$$

point: $(0,1)$

eqn of tangent line: $y-1 = \frac{1}{2}(x-0)$

$$y = \frac{1}{2}x + 1 \,//$$