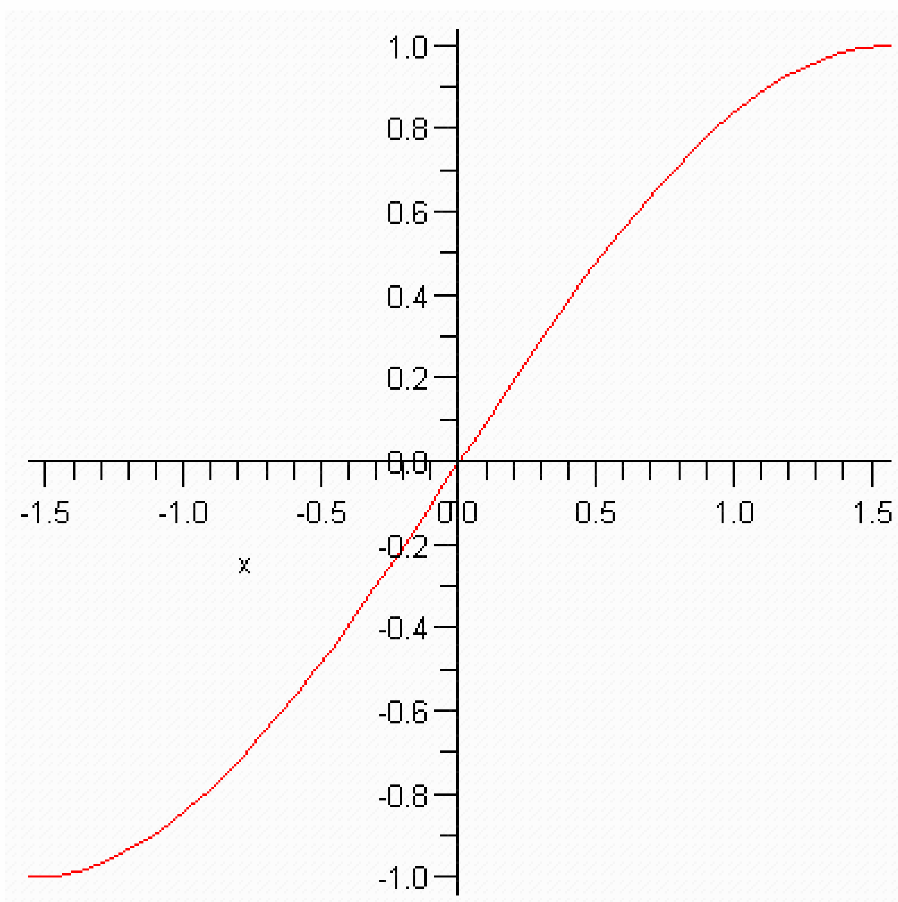
Demo on Linearizations.

f(x) = sin(x), at x=0

> $f := x \rightarrow sin(x)$

$$f := x \rightarrow \sin(x)$$

> $plot\left(f(x), x = \frac{-Pi}{2}..\frac{Pi}{2}\right)$



>

Find the linearization.

> $g := x \rightarrow D(f)(x)$

$$g := x \rightarrow (\mathrm{D}(f))(x)$$

> $eval(g(x))$

$$\cos(x)$$

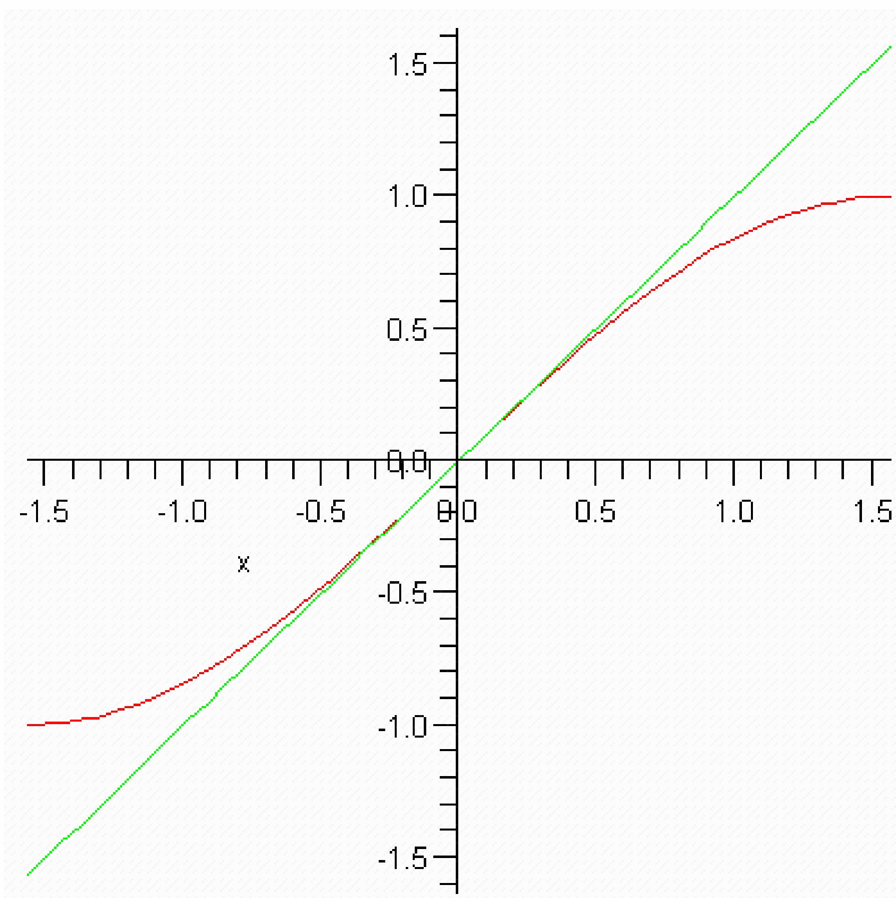
> $L := x \rightarrow f(0)$
> $\quad + g(0) \cdot (x - 0)$

$$L := x \rightarrow f(0) + g(0)\, x$$

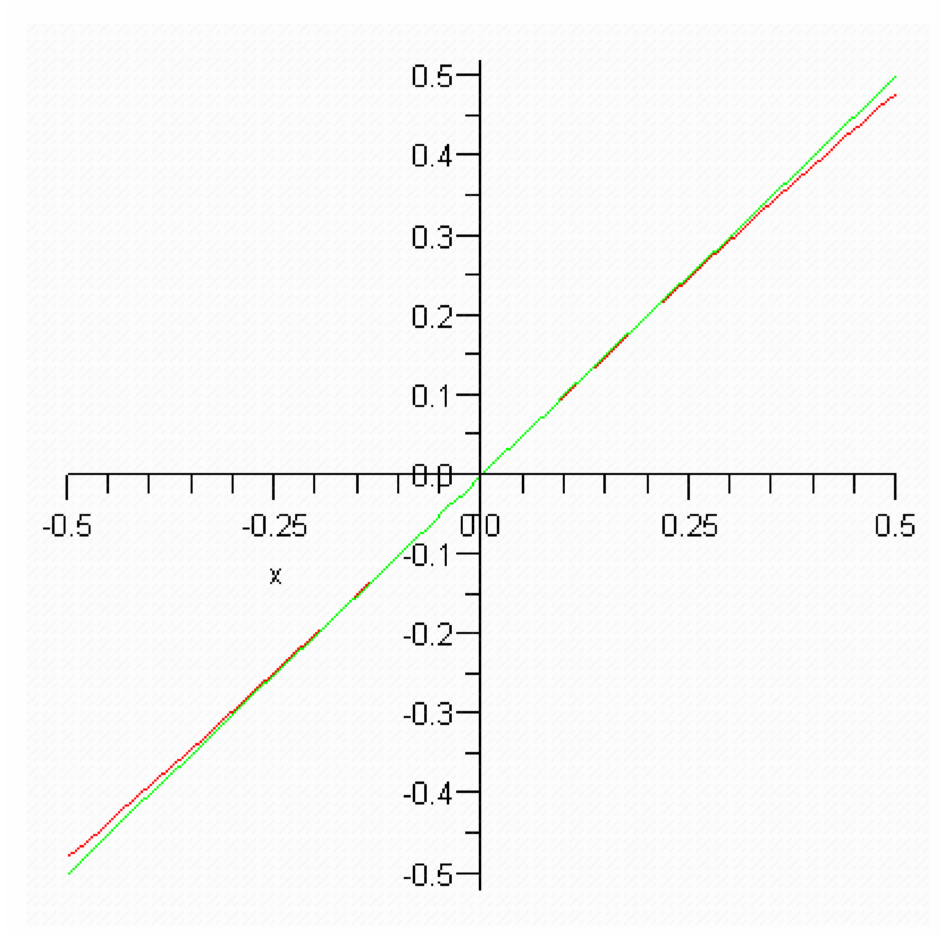
> $eval(L(x))$

$$x$$

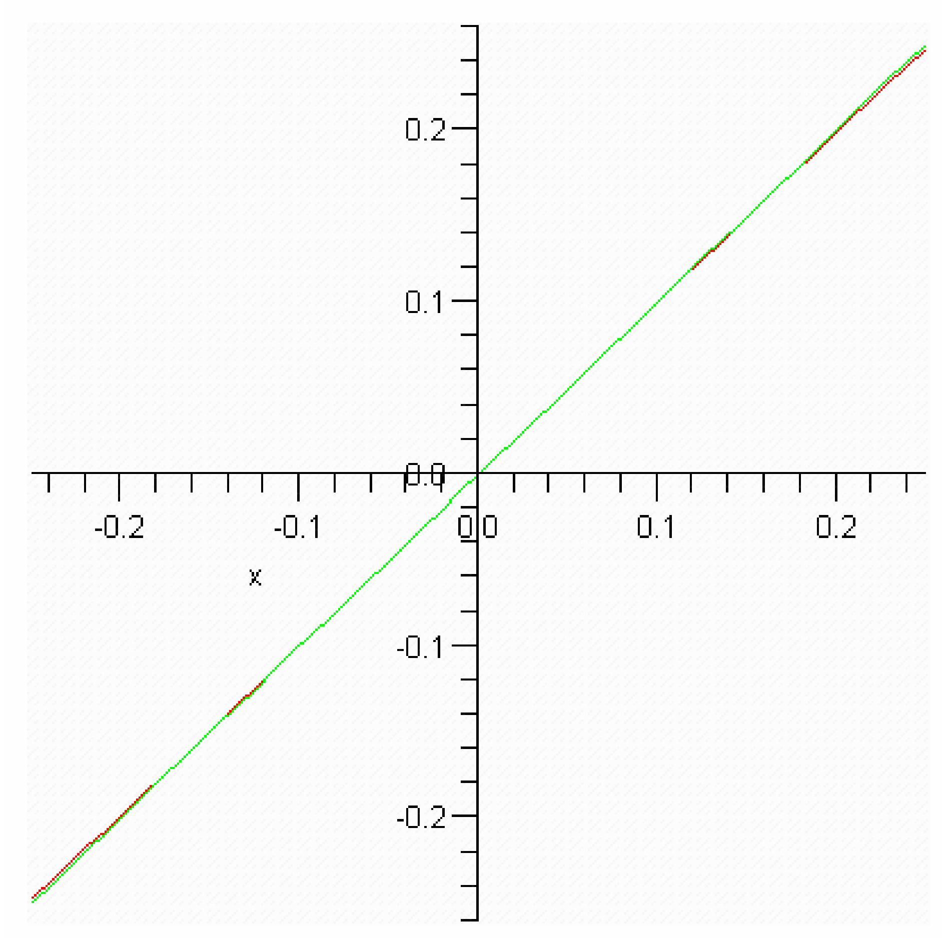
> $plot\left([f(x), L(x)], x = \frac{-Pi}{2} .. \frac{\pi}{2} \right)$



> $plot([f(x), L(x)], x = -.5 .. .5)$



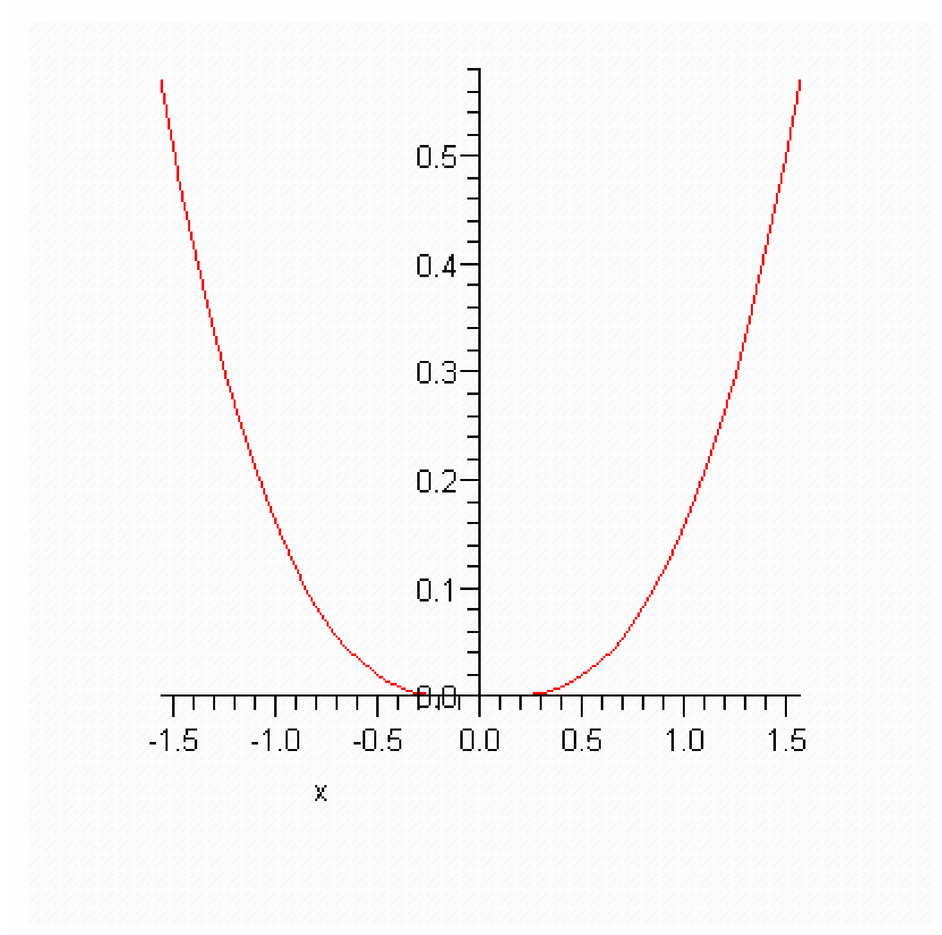
> $plot([f(x), L(x)], x = -.25 .. .25)$



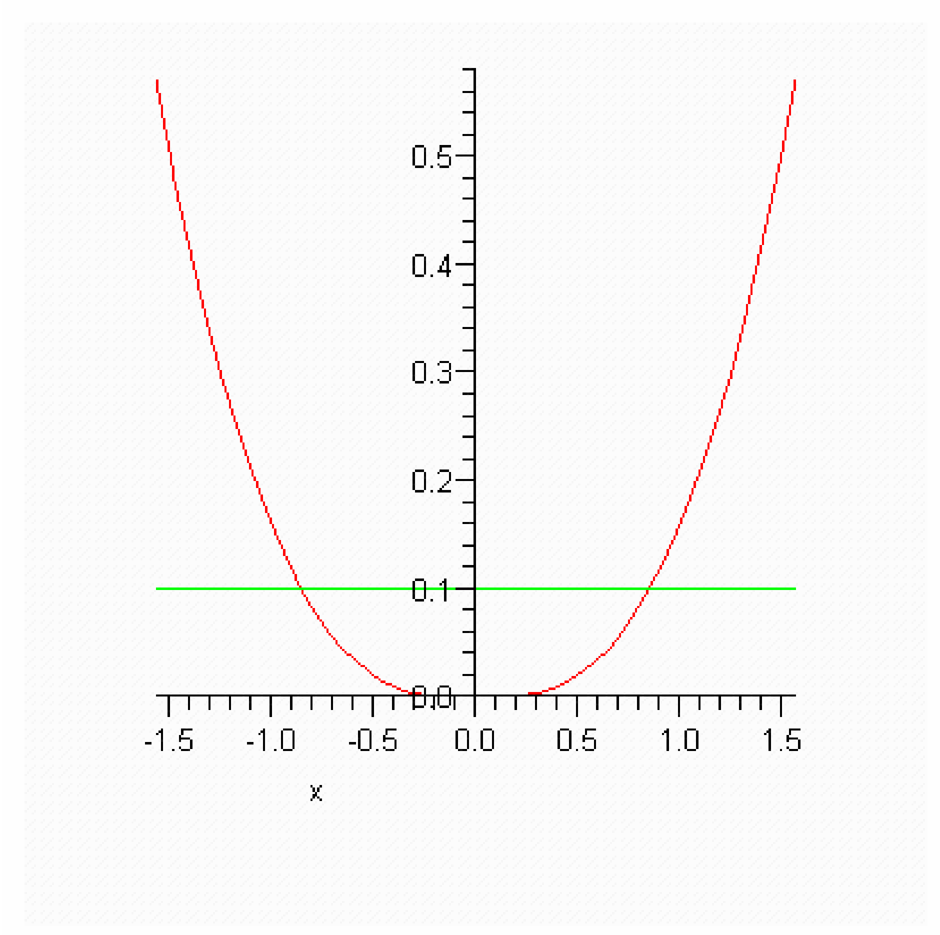
>

Look at the error.

> $plot\left(\left|f(x) - L(x)\right|, x = \frac{-Pi}{2}..\frac{Pi}{2}\right)$



> $plot\left([|f(x) - L(x)|, .1], x = \frac{-Pi}{2} .. \frac{\pi}{2} \right)$



>

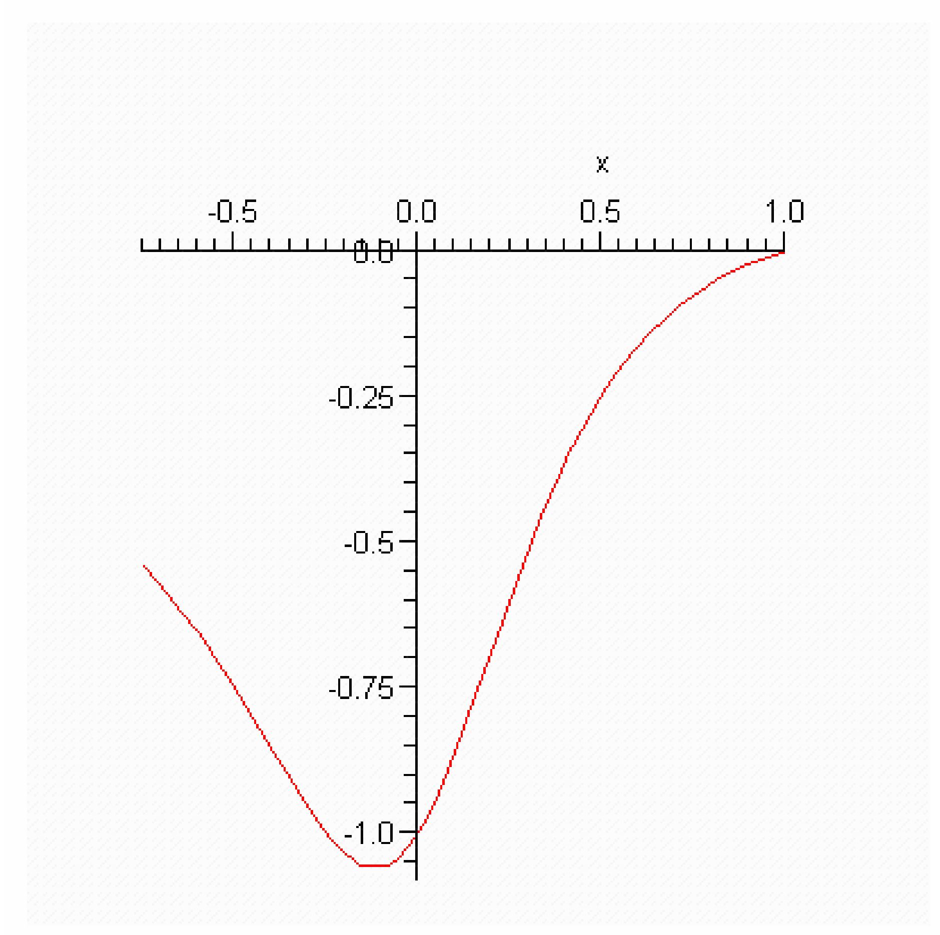
In order to guarantee that the error, |f(x)-L(x)|,is less than 0.1, it is enough that |x-0|<.8

Look at #80, p. 254.

> $f := x \rightarrow \dfrac{(x-1)}{(4 \cdot x^2 + 1)}$

$$f := x \rightarrow \frac{x-1}{4\,x^2+1}$$

> $plot(f(x), x = -.75\,..1)$



>

Linearize at x=1/2 (also say a=1/2).

> $g := x \rightarrow D(f)(x)$

$$g := x \rightarrow (\mathrm{D}(f))(x)$$

> $eval(g(x))$

$$\frac{1}{4\,x^2+1} - \frac{8\,(x-1)\,x}{(4\,x^2+1)^2}$$

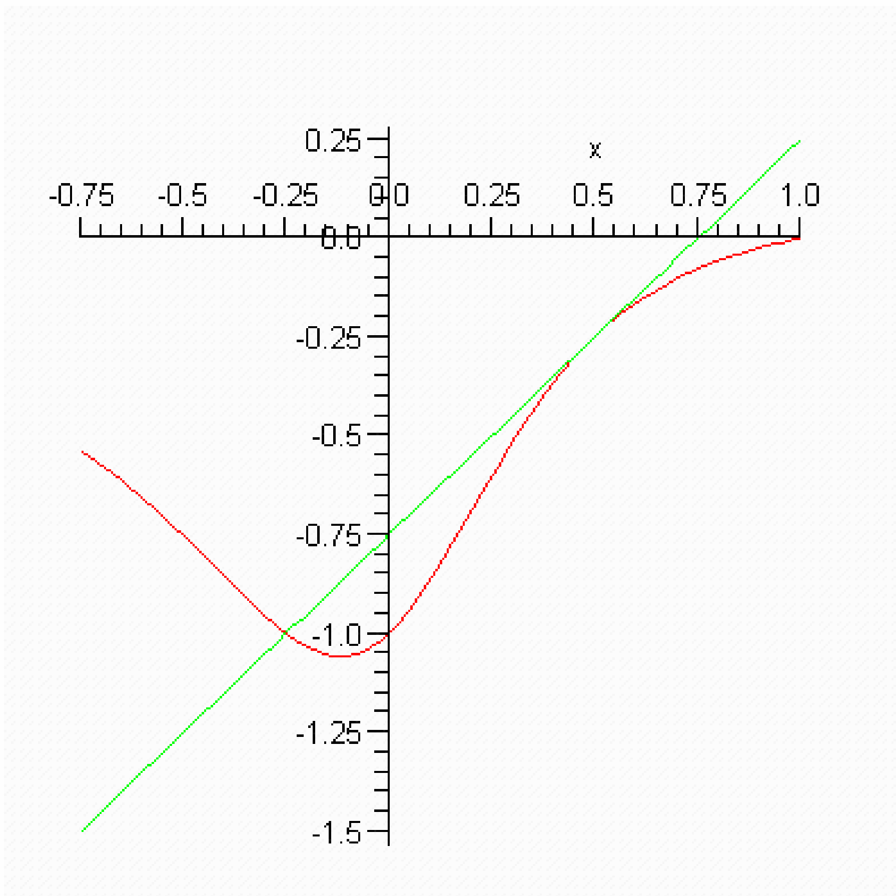
> $L := x \rightarrow f\left(\frac{1}{2}\right) + g\left(\frac{1}{2}\right) \cdot \left(x - \frac{1}{2}\right)$

$$L := x \rightarrow f\left(\frac{1}{2}\right) + g\left(\frac{1}{2}\right)\left(x - \frac{1}{2}\right)$$

> $eval(L(x))$

$$-\frac{3}{4} + x$$

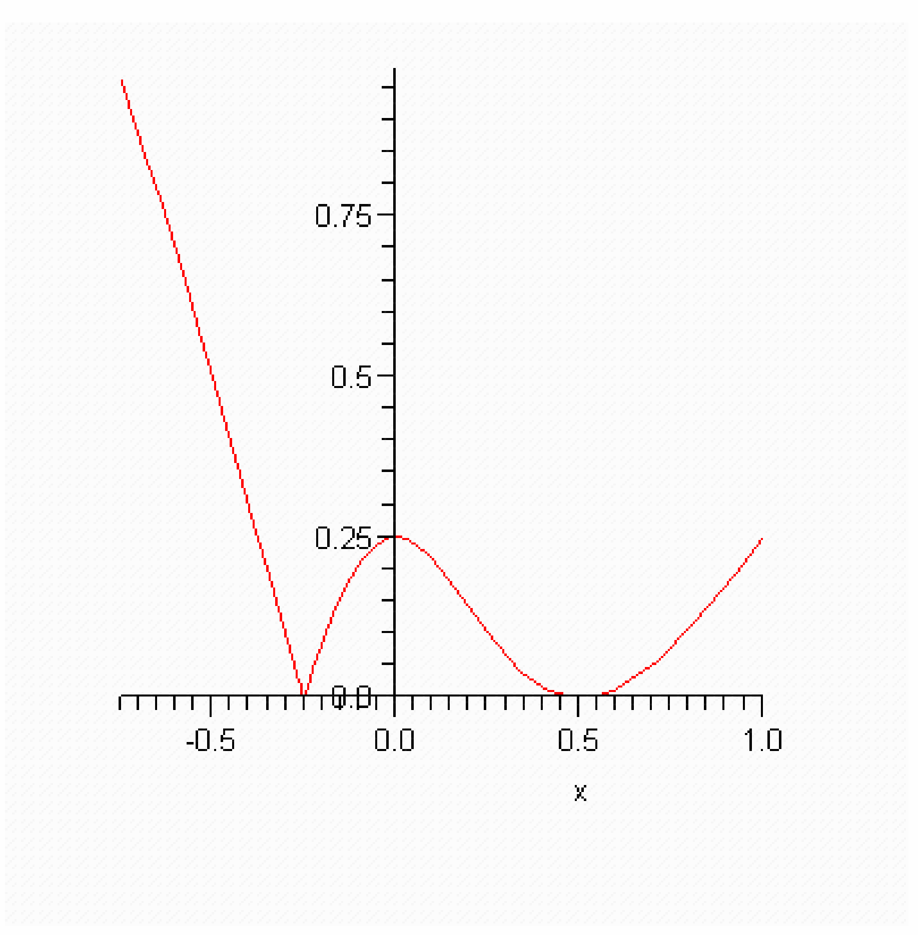
> $plot([f(x), L(x)], x = -.75 ..1)$



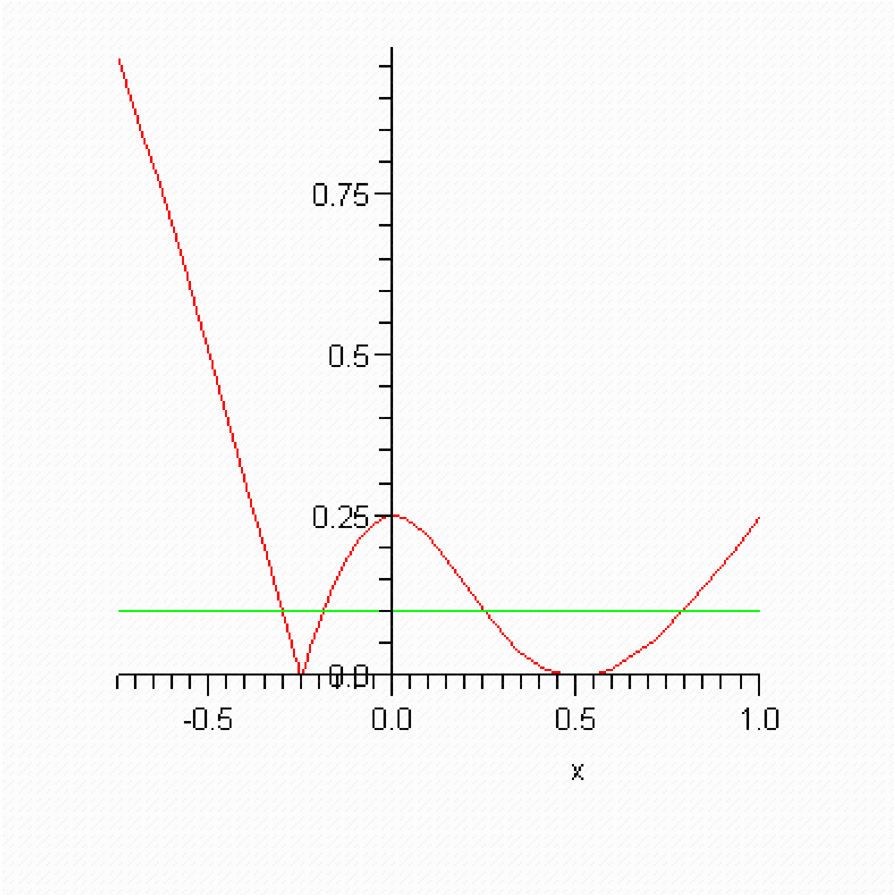
>

Look at error again.

> $plot(|f(x) - L(x)|, x = -.75 ..1)$



> $plot([|f(x) - L(x)|, .1], x = -.75 .. 1)$

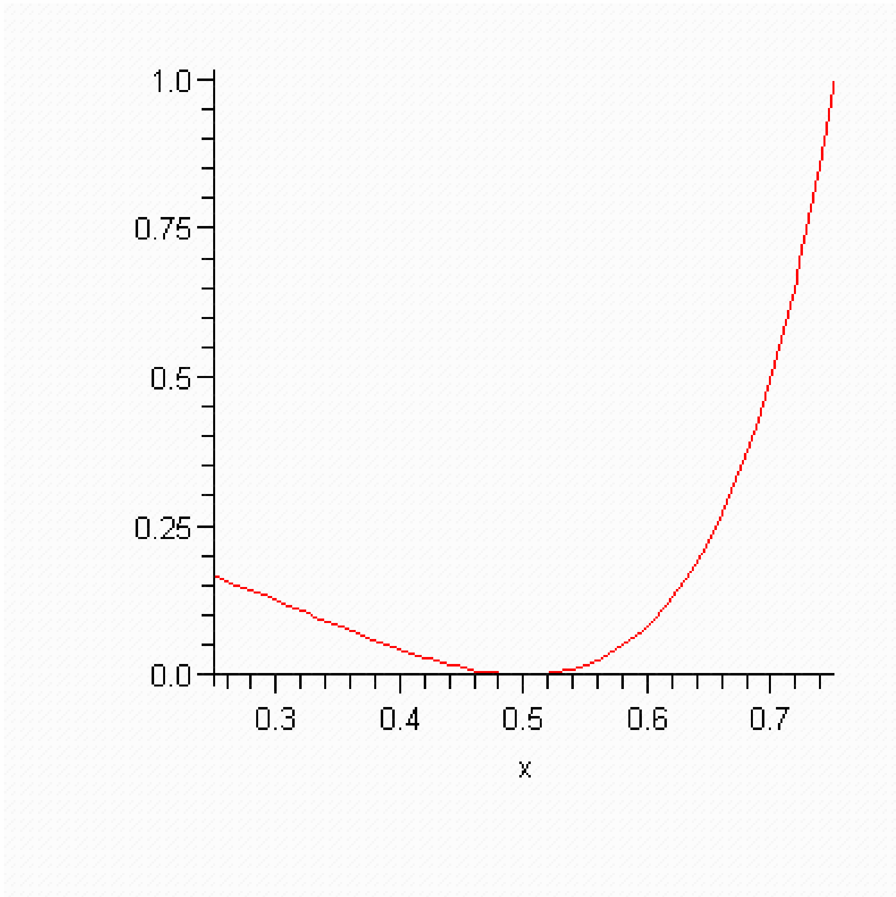


>

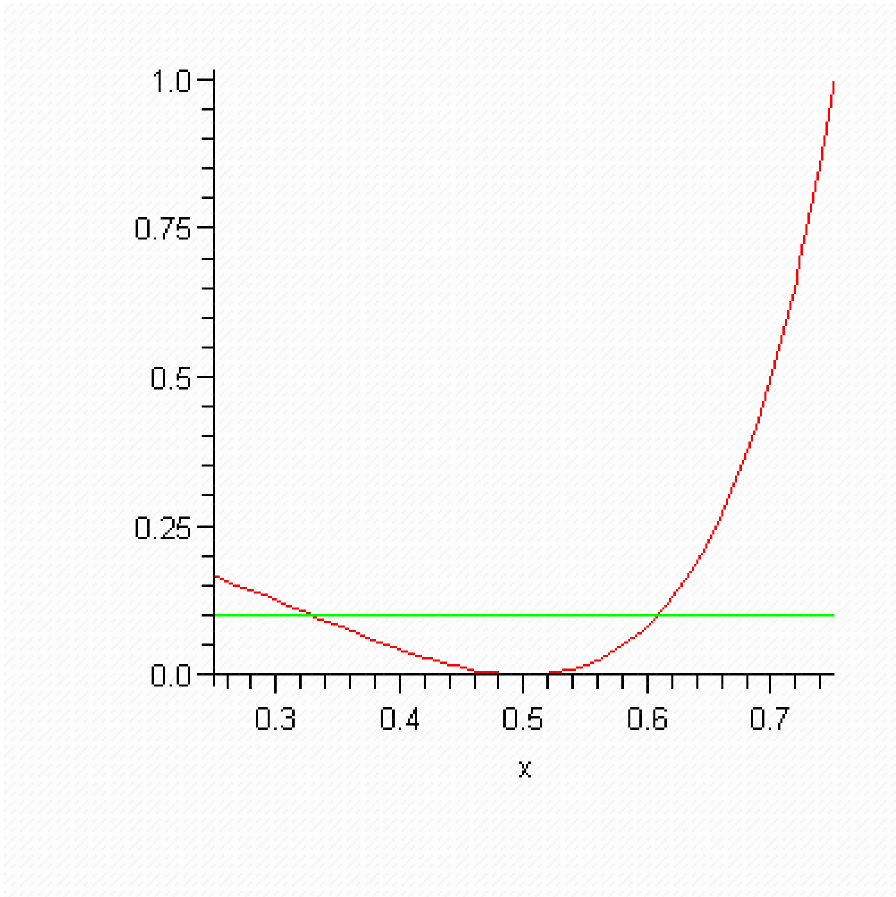
We can get the error |f(x)-L(x)|<.1 whenever |x-.5|<.25.

Can also look at percentage error (or relative error).

> $plot\left(\left|\frac{(f(x)-L(x))}{f(x)}\right|, x = .25 .. .75\right)$



> $plot\left(\left[\left|\frac{(f(x)-L(x))}{f(x)}\right|,.1\right], x=.25 .. .75\right)$



>

To guarantee that pct error is less than .1 (or 10%) it is enough that |x-.5|<.1