Approximating areas with rectangles.

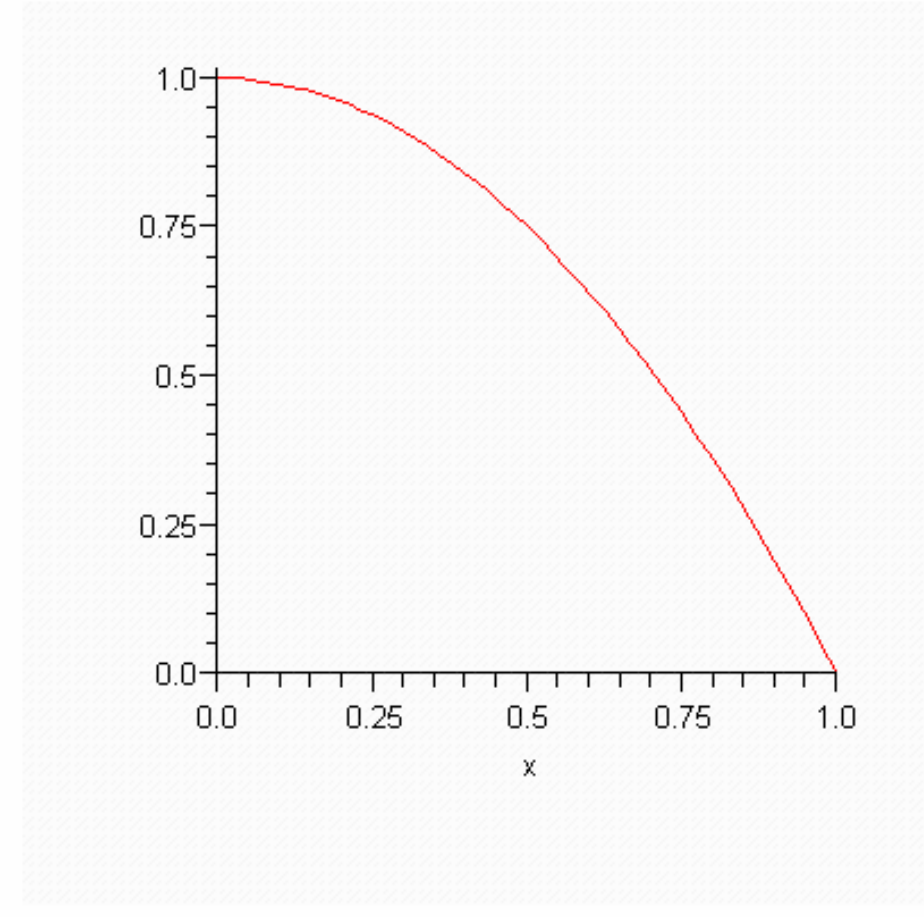
> $with(student)$

$[D, Diff, Doubleint, Int, Limit, Lineint, Product, Sum, Tripleint, changevar, completesquare, distance, equate, integrand, intercept, intparts, leftbox, leftsum, makeproc, middlebox, middlesum, midpoint, powsubs, rightbox, rightsum, showtangent, simpson, slope, summand, trapezoid]$

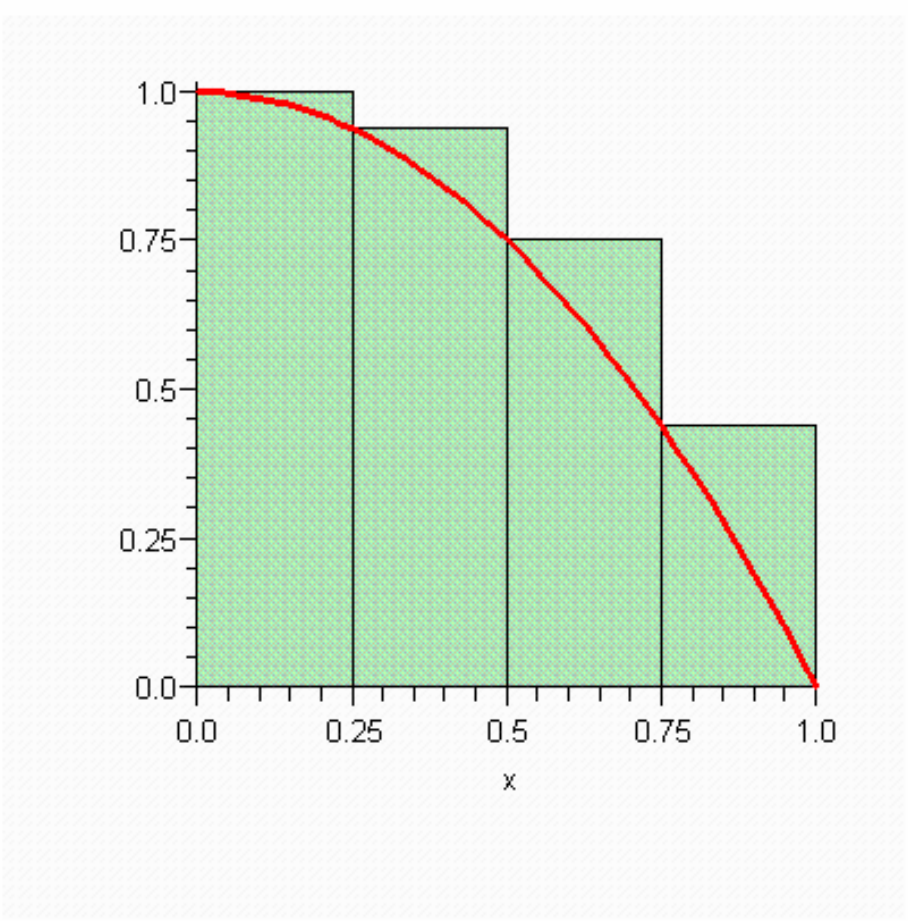
> $f := x \rightarrow 1 - x^2$

$$f := x \rightarrow 1 - x^2$$

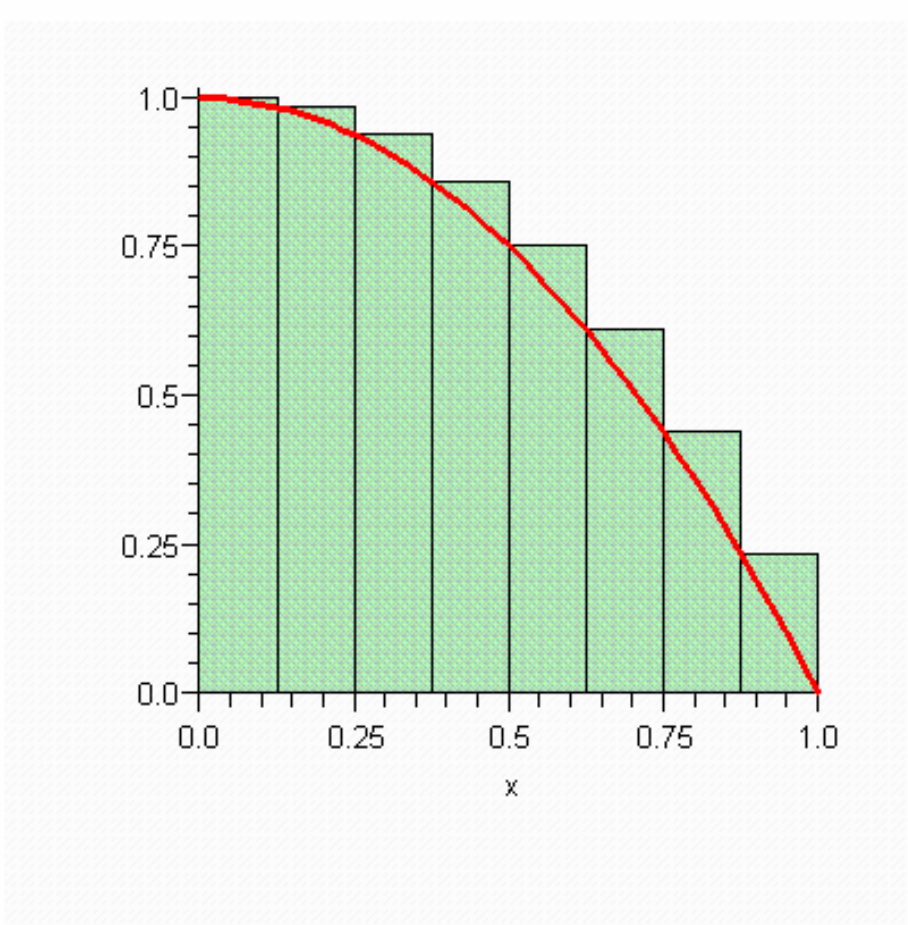
> $plot(f(x), x = 0..1)$



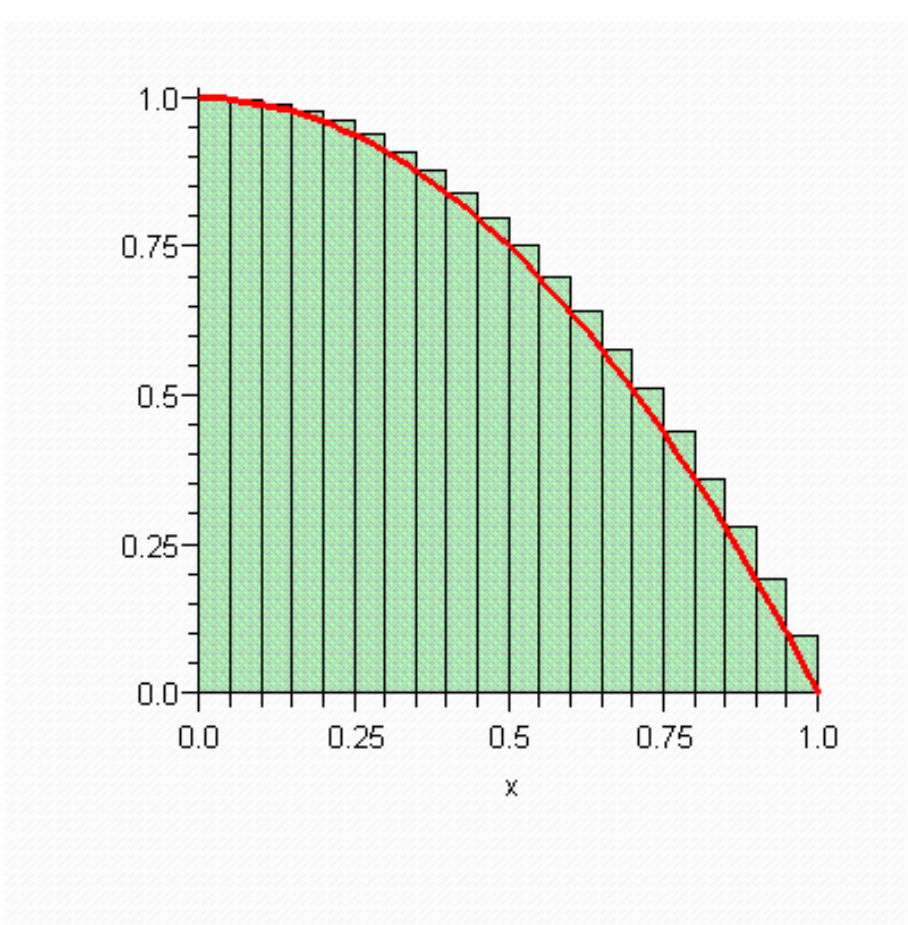
> $leftbox\,(f(x),\ x = 0\ ..1,\ 4)$



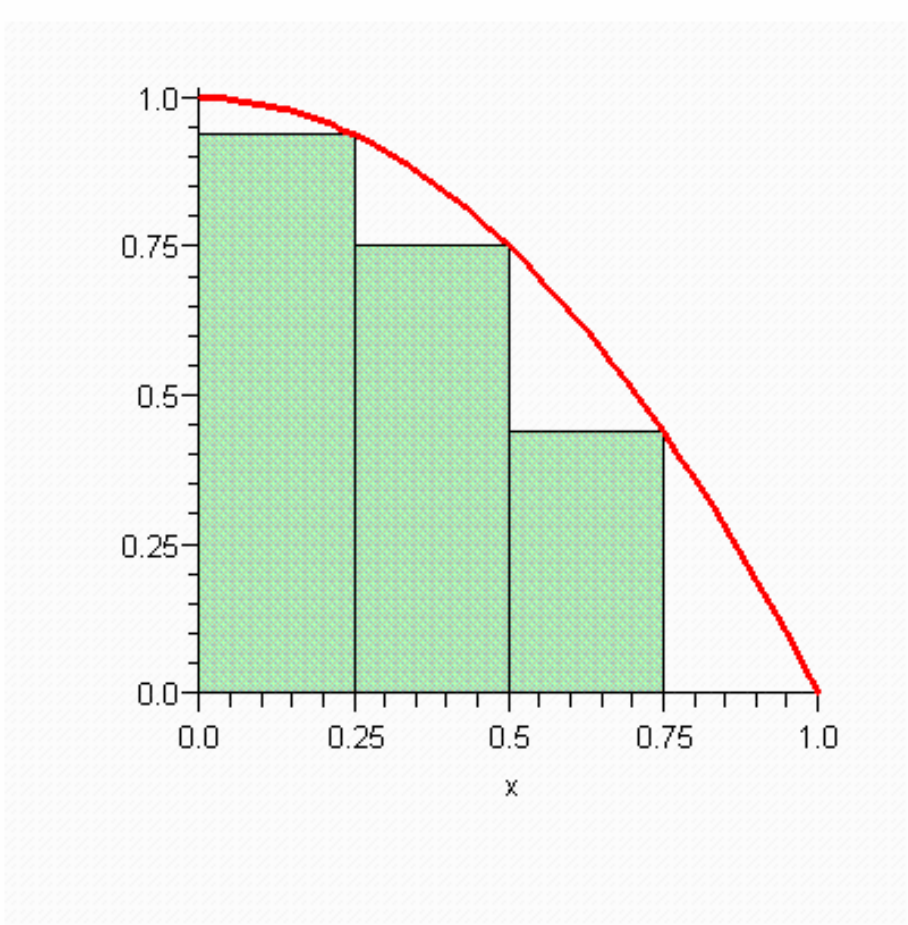
> $leftbox\,(f(x),\ x = 0\ ..1,\ 8)$



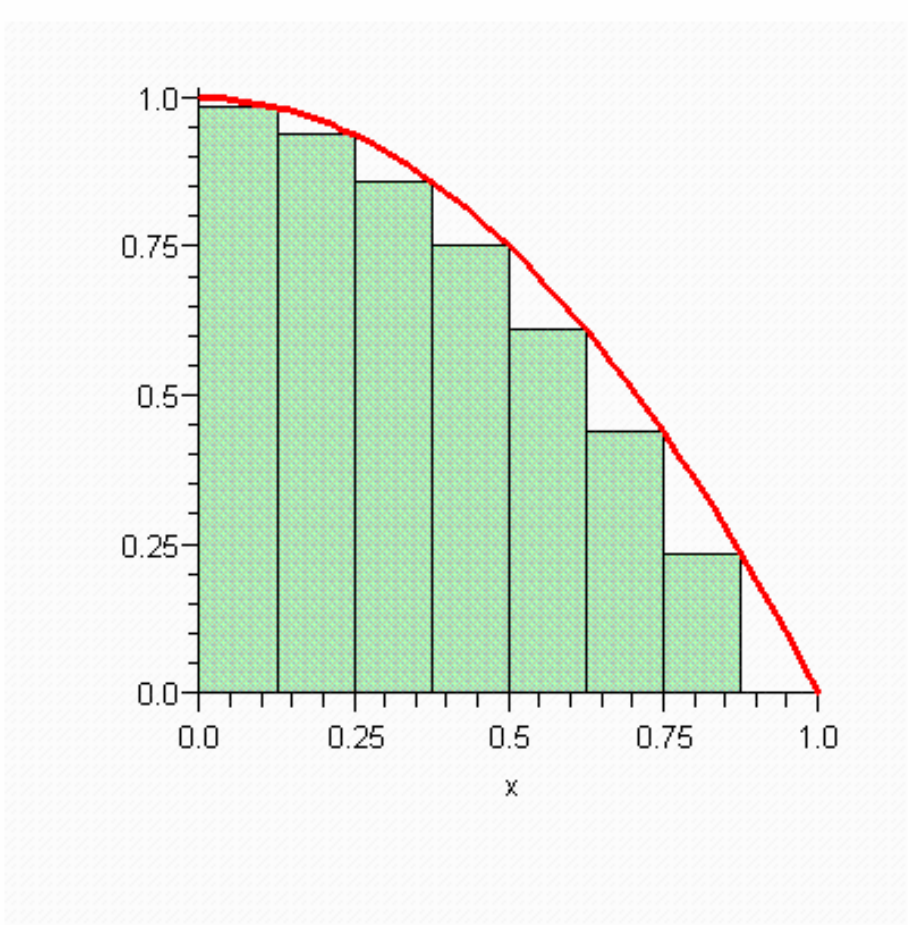
> *leftbox* $(f(x), x = 0..1, 20)$



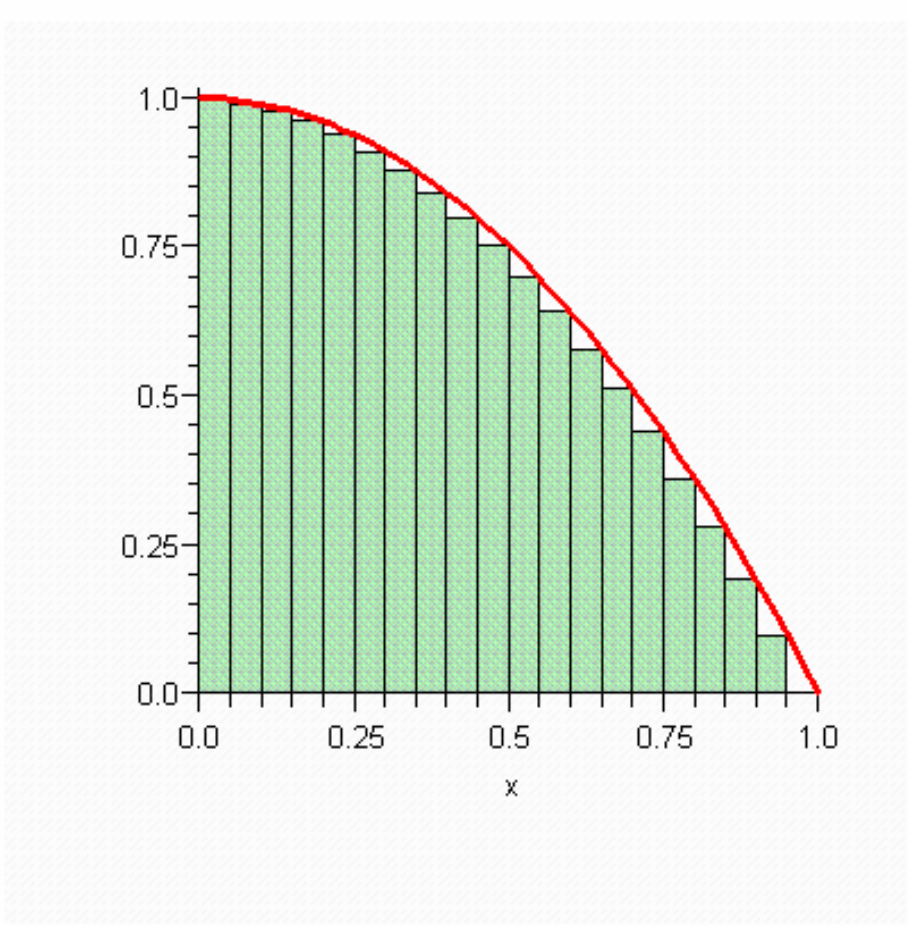
> $rightbox(f(x), x = 0..1, 4)$



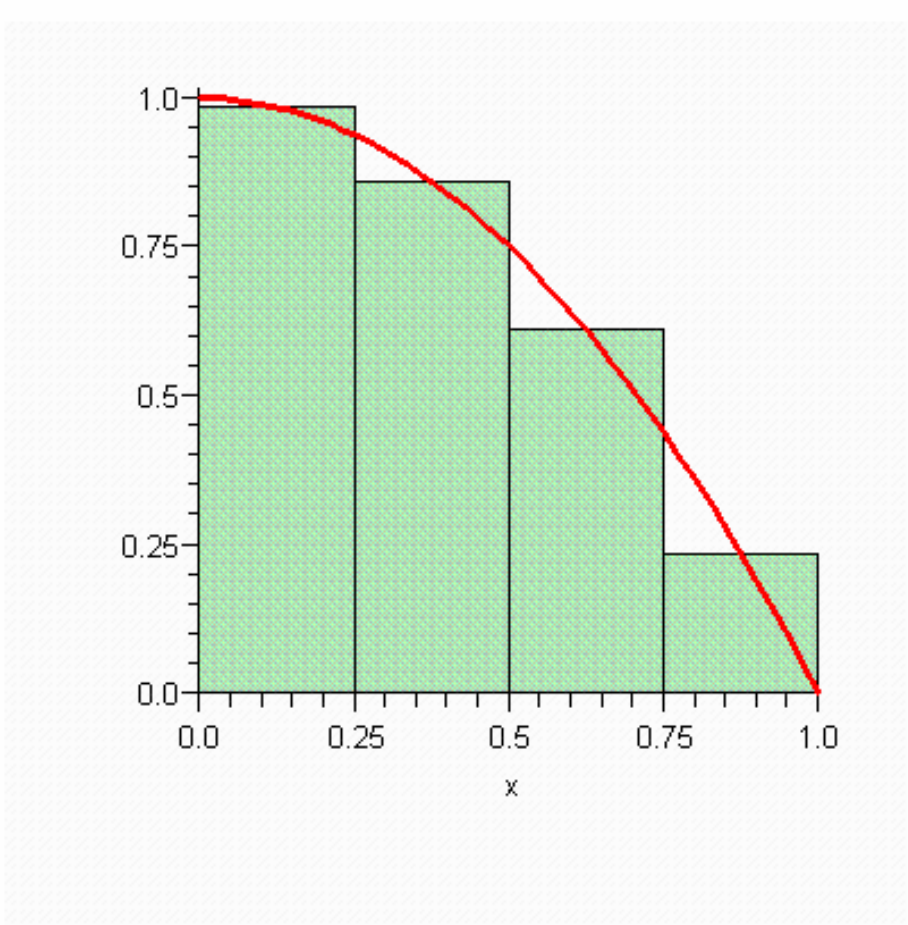
> $rightbox(f(x), x = 0..1, 8)$



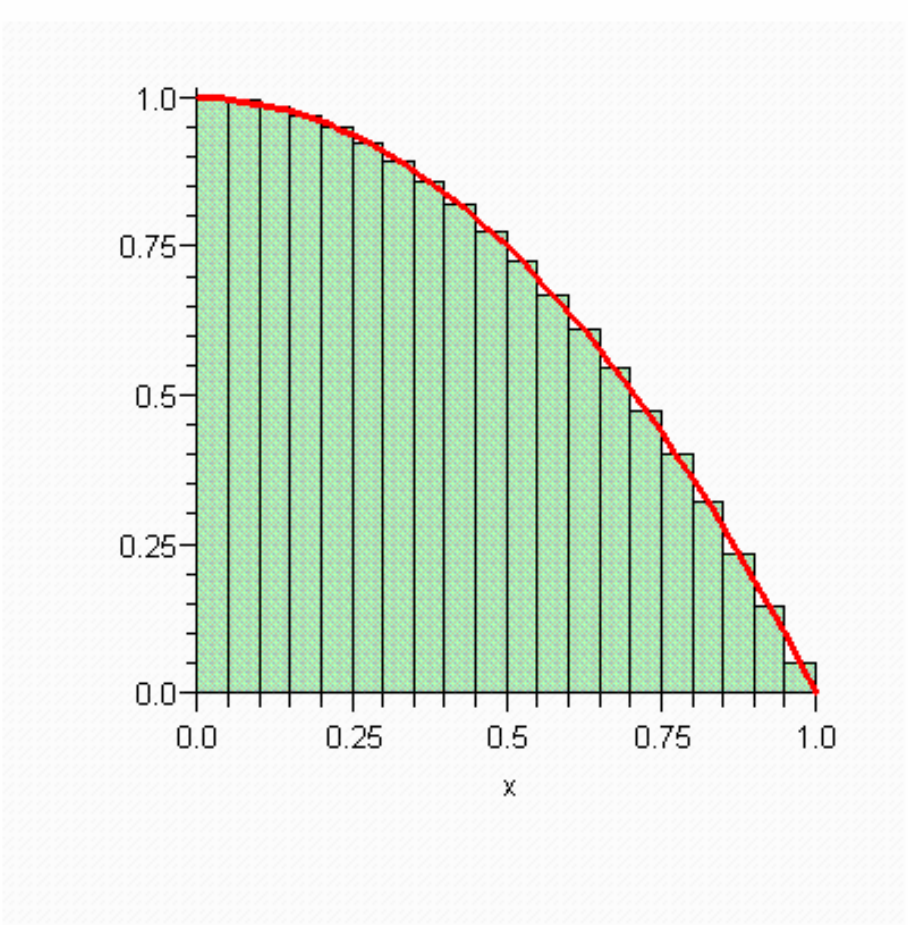
> $rightbox(f(x), x = 0..1, 20)$



> $middlebox(f(x), x = 0..1, 4)$



> $middlebox(f(x), x = 0..1, 20)$



> $leftsum(f(x), x = 0..1, 4)$

$$\frac{1}{4}\left(\sum_{i=0}^{3}\left(1-\frac{1}{16}i^2\right)\right)$$

> $leftsum(f(x), x = 0..1, 8)$

$$\frac{1}{8}\left(\sum_{i=0}^{7}\left(1-\frac{1}{64}i^2\right)\right)$$

> $leftsum(f(x), x = 0..1, 50)$

$$\frac{1}{50}\left(\sum_{i=0}^{49}\left(1-\frac{1}{2500}i^2\right)\right)$$

> $evalf(leftsum(f(x), x = 0..1, 4))$

$$0$$
$$.7812500000$$

> $evalf(leftsum(f(x), x = 0..1, 8))$

$$0$$
$$.7265625000$$

> $evalf(leftsum(f(x), x = 0..1, 20))$

$$0$$
$$.6912500000$$

> $evalf(leftsum(f(x), x = 0..1, 50))$

$$0$$
$$.6766000000$$

> $evalf(rightsum(f(x), x = 0..1, 4))$

$$0$$
$$.5312500000$$

> $evalf(rightsum(f(x), x = 0..1, 8))$

$$0$$
$$.6015625000$$

> $evalf(rightsum(f(x), x = 0..1, 50))$

$$0$$
$$.6566000000$$

> $middlesum(f(x), x = 0..1, 4)$

$$\frac{1}{4}\left(\sum_{i=0}^{3}\left(1-\left(\frac{1}{4}i+\frac{1}{8}\right)^2\right)\right)$$

> $middlesum(f(x), x = 0..1, 8)$

$$\frac{1}{8}\left(\sum_{i=0}^{7}\left(1-\left(\frac{1}{8}i+\frac{1}{16}\right)^2\right)\right)$$

> $evalf(middlesum(f(x), x = 0..1, 4))$

$$0.6718750000$$

> $evalf(middlesum(f(x), x = 0..1, 8))$

$$0.6679687500$$

> $evalf(middlesum(f(x), x = 0..1, 20))$

$$0.6668750000$$

>